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Comparative Study of Different Volume-Delay Functions in A Mathematical Programming based Dynamic User Equilibrium Traffic Assignment

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Abstract

A well-defined mathematical programming based approach is the most suitable form of Dynamic Traffic Assignment (DTA) for large-scale macroscopic transport models. If the mathematical programming based DTA uses Volume-Delay Functions (VDF) for travel time estimation, the transition from Static Traffic Assignment becomes more feasible. Theoretically, the assignment result of the DTA model should however depend on the performance of the particular VDF used. Therefore, the performances of four major VDFs (BPR, Conical, Akcelik and Logit-based) in a DTA model was investigated. Based on the large set of simulation data, a comparative analysis was performed with respect to 9 indicators. The results show that the performances of different VDFs vary widely with respect to different indicators. This suggests that the VDF needs to be chosen based on its performance in the most important indicators relative to a specific model.

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Declaration

I hereby confirm that the presented thesis work has been done independently and using only the sources and resources as are listed. This thesis has not previously been submitted elsewhere for purposes of assessment.

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Maksudul Alim

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1. Introduction

Dynamic Traffic Assignment (DTA) is a quite popular assignment method for microscopic and mesoscopic traffic simulation. However, for macroscopic large-scale transport models, DTA methods are still being overlooked in favor of Static Traffic Assignment (STA). Although DTA method offers superior modeling capability in terms of spatial-temporal dynamics and realistic traffic behavior (e.g., queue spillback and speed variations), yet DTA methods are considered less-favorable for long-term strategic transport planning because of their mathematical complexity, requirement of large set of input data and computational challenge. However, a particular type of DTA, known as the mathematical programming based DTA approach, can overcome those limitations because of its simplicity, well defined mathematical properties, less data requirement and better computational efficiency. Such a mathematical programming based DTA method is available in a commercial travel demand modeling software package called TransCAD. This particular DTA method is a Dynamic User Equilibrium (DUE) based approach and dependent on Volume-Delay Functions (VDF), same as traditional STA. By using VDF for dynamic travel time calculation, this DTA method offers a feasible alternative to the STA for the long-term strategic transport planning.

Boyce et al. (2001, p. 386) implied that the performance of a mathematical programming based DUE is heavily influenced by the particular VDF applied. This suggests that different VDF perform differently in a DTA model. Therefore, it was felt quite intriguing to conduct a research on the comparative performances of different VDFs in a DTA model. Almost all of the researches conducted in the relevant area at past, focused only on the performance of VDFs in STA model (explained later in *chapter 2.5*). Therefore, it was understood that there is still room for more investigations on the performance of VDFs in DTA model. The underlying question this research focused is “*how do different VDFs perform comparatively in a DTA model?*”

Using the TransCAD software package, a DTA model was developed for the Munich city, as a case study. The urban traffic of the Munich city area enabled the DTA to be performed at the peak hours, when it is likely to produce congestion. The congested traffic condition, particularly around the intersections, presents some interesting possibilities in terms of the performance of different VDFs. Four VDFs (BPR, Conical, Akcelik and Logit-based) have been tested in this research, among which the latter two directly depend on the intersection delay. This research aimed at studying the performance of these four VDFs in the macroscopic DTA model with respect to different indicators and model properties.

This thesis paper is structured in following outline:

- Chapter 2 covers sequential and brief reviews on relevant topics, starting from the DTA method to the four VDFs used in this study. The chapter is concluded with a review of the previous comparative studies on VDF.
- Chapter 3 offers an overview on the algorithm of the particular DTA method used in this research.
- Chapter 4 explains various component of the model and describes relevant datasets and model specifications.
- Chapter 5 explores the results obtained from the comparative analysis of different VDFs.
- Chapter 6 acknowledges the limitations, recommends the scopes for further research and finally concludes the thesis by summarizing the outcome.

2. Literature Review

2.1. Dynamic Traffic Assignment

Traffic assignment methods have historically been focused on Static Traffic Assignment (STA) which represents, according to Transportation Research Board (2011, p. 7), the "average or steady-state traffic conditions over an analysis time period that is long compared to the time scale of traffic dynamics". However, there are a number of widely recognized limitations of STA models. Wang et al. (2018, p. 371) described it as "the incapability of modeling realistic traffic dynamics (e.g., queue spillback, and speed variations), as well as the spatial and temporal vehicular interactions". On the other hand, Dynamic Traffic Assignment (DTA) represents variations in traffic flows and conditions over the analysis period and therefore attempts to reflect the reality that traffic networks are generally not in a constant or steady state. Szeto and Lo (2006, pp. 48–49) noted that DTA models are regarded as capable of replicating more realistic traffic flow characteristics while some of the more advanced ones are even able to simulate shockwaves, queue formulation and dissipation, queue spillback, lane changing behavior, hysteresis phenomenon, etc.

DTA was primarily developed in the 1970s. The first instance of DTA can be dated back as early as the initial work of Samuel (1971), however it is the pioneering contribution of Merchant and Nemhauser (1978a, 1978b) that played an influential role into the later development of DTA. Since then DTA has been of a great interest of the researchers, as Szeto and Wong (2012, pp. 3–4) noted that the number of DTA-related publications increased quite significantly from 1993 onwards.

As a result of vast researches and publications on DTA, several literature review studies have been undertaken in order to synthesize the numerous research developments and outline the future research scopes. Cascetta and Cantarella (1993) took the first attempt to review DTA related literatures published before 1991. Afterwards, Peeta and Ziliaskopoulos (2001) conducted a very comprehensive and thorough review of DTA models and methodologies developed till 2000. In the same year, Boyce et al. (2001) analyzed the analytical DTA solution methods and particularly focused on the variational inequality formulation. In the year of 2005, Szeto and Lo carried out two review studies in two different directions. Szeto and Lo (2005a) compared DTA properties in terms of different forms of traffic flow models and addressed their implications. On the other hand, Szeto and Lo (2005b) studied the DTA properties with and without considering the effects of spatial queues, as well as discussed their implications. Lately, Mun (2007) provided a review of the traffic flow performance of DTA models and identified the strength and weakness of existing models.

Almost all relevant DTA literatures, such as Peeta and Ziliaskopoulos (2001, pp. 234–235), Janson et al. (2001, pp. 353–354), Szeto and Wong (2012, p. 13), Saw et al. (2015) unanimously classified DTA models into two broad methodological groups, which are analytical-based DTA and simulation-based DTA. Correspondingly, analytical-based DTA models are then further categorized into three following sub-groups:

1. Mathematical programming-based DTA
2. Optimal control-based DTA
3. Variational inequality-based DTA

However, in addition to above 3 approaches, Szeto and Wong (2012, p. 13) included three further analytical DTA categories:

4. Nonlinear Complementarity Problem -based DTA
5. Fixed-Point Problem-based DTA
6. Continuum Modeling Problem-based DTA

Most leading researchers in the DTA field such as Ben-Akiva et al. (2012, p. 63), Carey (2001, p. 350) and Peeta and Ziliaskopoulos (2001, p. 240) agreed that the simulation-based DTA is the best suited method for realistic traffic modeling due to its microscopic traffic flow characteristics and its ability to model detailed traffic dynamics as well as wide range of operational strategies. However, the simulation-based DTA has some notable disadvantages when compared to the analytical DTA and especially to the mathematical programming approach. Ben-Akiva et al. (2012, pp. 63–64), Janson et al. (2001, p. 354) and Szeto and Wong (2012, p. 12) identified computational inefficiency, lack of proper mathematical formulations and skepticism about reaching convergence as the major limitation of simulation based DTA.

On the other hand, Boyce et al. (2001, pp. 377–378), Janson et al. (2001, p. 354) and Carey (2001, p. 351) have pointed out some exclusive advantages of analytical DTA such as well-defined mathematical properties and ability to confirm when the convergence is reached. Analytical DTA considers traffic behavior at the aggregate level, i.e., macroscopic scale and produces a dynamic version of the equilibrium principle of Wardrop (1952). However, Szeto and Wong (2012, pp. 12–13) and Janson et al. (2001, pp. 354–355) stated that having already quite complicated formulations, analytical DTA can only offer very limited functionality in terms of simulating the complexity of traffic flow dynamics.

2.2. Mathematical Programming based DTA

Mathematical programming based DTA approach is also known as optimization method. Mathematical programming based DTA models formulate the DTA problem by splitting the assignment period into discrete time-intervals. The earliest instance of mathematical program based DTA model was the system optimizing model of Merchant and Nemhauser (1978a, 1978b). This model is widely referred as M-N model. The model propagates traffic by using a link exit function and the travel cost is represented by a volume-delay function. It results in a flow-based, discrete time, non-convex non-linear programming formulation. Peeta and Ziliaskopoulos (2001, pp. 235–237) referred to the M-N model as deterministic, fixed-demand, single-destination and single-commodity System Optimal (SO) assignment. The global solution of the M-N model's assignment problem requires solving a piecewise linear version of the model. Furthermore, Ho (1980) proved that the maximum number of such linear pieces needs to be solved sequentially is "the number of time intervals+1".

The nonconvex nature of M-N model is caused by the nonlinear constraints. Carey (1986) demonstrated that the M-N model fulfills the linear independence constraint qualifications, which ensured that the necessary optimality conditions would hold at an optimum. Consecutively, Carey (1987) introduced inequality in the link exit function, which transformed the M-N model into a convex problem. With the help of this improvisation, the convex M-N formula gained mathematical and algorithmic suitability. As a result, standard mathematical programming software can be used to solve the problem. Further down the road, Carey (1992) revealed that the basic convex M-N model, which was still a single-destination and single-commodity assignment problem, can be extended to accommodate multiple destinations and commodities. However, any extension from single-destination to multiple destinations requires the model to explicitly satisfy a ``First-In, First-Out'' (FIFO) condition. FIFO condition means traffic which embarks on a road or other facility in a particular time period, exits from that facility ("on average") before other traffic that enters in any later time periods. This FIFO requirement would introduce additional constraints which make the resulting formulations once again non-convex, thus computationally less tractable. According to Peeta and Ziliaskopoulos (2001, pp. 235–237), such FIFO associated non-convexity issue is inherent in all mathematical programming based DTA approaches, for both the UE and SO cases.

Few years later, Carey and Subrahmanian (2000) established another issue associated with mathematical programming based models called "holding-back". "Holding-back" refers to a phenomenon when the travel time of a link increases along with the increasing flow rate of that link to an extent so that it may be convenient to "hold back" traffic from entering that link, in order to minimize overall travel times or costs. Therefore, "holding-back" scenario holds traffic back at the upstream link until congestion on downstream links is lower. Peeta and Ziliaskopoulos (2001, pp. 235–237) noted that from a modeling standpoint, incorporating "hold back" issue into the model, would prompt the need for additional constraints.

Prior to Janson (1991a, 1991b), all research work in the field of mathematical programming based DTA considered the assignment problem as SO formulation. His pioneering contribution is regarded as one of the earliest efforts into the formulation of Dynamic User Equilibrium (DUE). Another notable feature of his DTA approach is that it seeks an equilibrium condition based on the experienced path travel times, instead of the instantaneous travel times assumed in several prior studies. The DUE is a temporal extension of the existing Static User Equilibrium (SUE) assignment problem with additional constraints to insure temporally continuous paths of flow. However, these flow constraints are non-linear in DUE, in contrast to SUE case in which the assumption of steady-state flow allowed all constraints to remain linear. Therefore, if the typical solution for SUE (the linear combinations method) is applied to DUE, it makes the assignment flow temporally discontinuous. This DUE method of Janson, is a heuristic approach that generates approximate solutions to DUE in an efficient manner for large networks. However, Peeta and Ziliaskopoulos (2001, pp. 235–237) suspected that the properties of Janson's procedure are not sufficiently well-established, and it may lead to possible unrealistic traffic behavior.

2.3. Dynamic User Equilibrium (DUE)

The equilibrium condition of a traffic network was primarily introduced by Wardrop (1952) through two of his principles. Wardrop's first principle is widely known as User Equilibrium (UE) and the second principle is referred to as System Optimal (SO). In this sub-chapter only UE will be highlighted and the focus will gradually be narrowed down to DUE.

UE establishes the equilibrium at the user level. From an available set of routes between origin to destination (O-D), the user chooses the route with the least travel time (or any other cost/disutility). However, the travel time or cost of a route between an O-D also depends on the choices made by other users, who are themselves also trying to choose the least travel time route between their own O-D. When every user succeeds in finding such a route which has the minimum travel time between O-D; moreover, for each O-D pair, every route used has the same travel time. Transportation Research Board (2011, p. 6) defined this condition as user equilibrium (UE).

Regarding the mathematical formulation of UE, Iryo (2013, p. 53) noted that UE is formulated by a system of equations that describe the conditions of equilibrium, rather than a certain algorithm. Beckmann et al. (1956) were the first to rigorously formulate the equilibrium conditions mathematically. They formulated the user equilibrium assignment problem as a nonlinear optimization problem. However, such nonlinear programming problem is computationally very difficult to solve, especially for a realistically sized network, as per Rakha and Tawfik (2009, p. 9436). Therefore, Frank and Wolfe (1956) introduced a much simpler linear approximation in order to replace the nonlinear problem, although this iterative linearization procedure still lack the decent computational efficiency. Consecutively, LeBlanc et al. (1974) presented an equilibrium solution technique for a mathematical programming based model that can converge more efficiently and closes in on the equilibrium flows rapidly without excessive computational requirements.

On the subject of UE, it is necessary to discuss instantaneous and experienced travel time with each of them implying different behavioral assumption. Experienced or actual travel time is defined as the travel time actually experienced by users during the trip. In traffic assignment models based on experienced travel time, users are assumed to have full and perfect knowledge of future traffic conditions at the time of their departures. Therefore, experienced travel time based UE model is also referred to as the predictive UE model. On the other hand, Instantaneous travel time is defined as the travel time based on the currently prevailing traffic conditions. The instantaneous travel time based UE model is also known as reactive UE model. Static UE models are typically reactive UE models based on instantaneous travel time.

Dynamic User Equilibrium (DUE) is a special case of UE. Transportation Research Board (2011, pp. 7–8) implied that the classical static UE requires two distinct extensions in order to formulate a DUE solution. First, instantaneous travel time based reactive UE model extends to the experienced travel time based predictive UE model. In other words, the assumption of perfect traveler information required in static UE is replaced by another assumption that users know or at least anticipate future travel conditions along the journey given that the travel times on dynamic network vary over time. Secondly, in a dynamic approach, the UE condition of equal travel times on used routes applies only to users who are assumed to depart at the same time between the same O-D pair.

Ran and Boyce (1996, p. 91) provided a convincing definition of ideal DUE in following words: *"If, for each O-D pair at each interval of time, the actual travel times experienced by users departing at the same time-interval are equal and minimal, the dynamic traffic flow over the network is in a travel-time-based ideal dynamic user-optimal state."*

2.4. Volume-delay Function (VDF)

A key element of a UE algorithm is the calculation of experienced or actual travel time throughout the network. The actual travel time of each link is calculated based on the congestion level using Volume-delay Function (VDF), which is also known as link performance function. VDF calculates and updates the link travel times iteratively during UE process. VDF functions are mathematical descriptions of the relationships between travel time and link volume. Almost all VDF follow the basic principles of traffic flow theory, which is if volume increases relative to capacity, speed decreases and link travel time increases.

According to Ortúzar and Willumsen (2011, p. 352) and Spiess (1990, p. 155), VDF needs to fulfill some mathematical and behavioral conditions in order to guarantee convexity and therefore convergence at during UE process. The VDF function must be non-decreasing, positive, continuous, differentiable and properly defined for oversaturated condition.

2.4.1. Bureau of Public Roads (BPR)

BPR function is the most popular VDF and very well suited for application with traffic assignment models. This function can represent a wide variety of flow-delay relationships by changing its parameter values.

BPR function was first developed by the Bureau of Public Roads (1964), which was a predecessor of Federal Highway Administration. The BPR function describes the link travel time as a function of the volume/capacity ratio.

$$t(v) = t_0 \left[1 + \alpha \left(\frac{v}{c}\right)^{\beta}\right]$$

where:

t = Link travel time
t_0 = Free flow travel time
v = Traffic volume
c = Link Capacity
α = Parameter
β = Parameter

The appropriate values for α and β parameters are discussed in *chapter 4.6.1*.

There are many advantages of BPR function which explains the wide popularity of BPR. Community Planning Association of Southwest Idaho (2017, p. 44) credited the simplistic form of BPR and also mentioned that it works well till the V/C ratio is less than 1.2. Moreover, Spiess (1990, p. 153) showed that when traffic volume equals the capacity, the speed becomes half of the free flow speed irrespective of α and β parameter values, which is very convenient.

However, the BPR function sometimes does not perform very well under certain traffic conditions. Dowling et al. (1997, p. 68) and Dowling and Skabardonis (1997, pp. 19–20) evaluated the BPR curve against the 1994 Highway Capacity Manual data and discovered that the BPR function tends to underestimate speeds when V/C ratios between 0.80 and 1.00 and overestimate speeds in severely congested conditions (when V/C >1). Dowling et al. (1997, p. 1) also hinted that the possible reason for such unrealistic behavior of BPR function during congestion could be explained by the fact that BPR function was originally developed by fitting into the uncongested freeway data from the late 1950s or early 1960s. In any case, this problem is particularly relevant to this research, since it involves a DTA model simulating the peak hour condition, which is supposed to yield congested traffic. In order to fix this issue, Dowling et al. (1997, p. 1) suggested to use facility-specific values of free-flow speed and capacity. In his opinion, it “cuts the error of the BPR technique in half”. The facility specific free-flow speed and capacity values used in this research are presented in *chapter 4.4*.

Moreover, Spiess (1990, p. 153) warned that even at uncongested condition, the V/C ratio of BPR function could be very high during the first few iterations. This will eventually lead to the need of high number of iterations and thus result a slower convergence.

Considering all these limitations of BPR function, different types of VDF such as Akcelik, Conical, Logit-based etc. functions have been developed over the years.

2.4.2. Conical Volume-Delay Function

The Conical function was developed by Spiess (1990). He demonstrated the computational deficiency of BPR function and then proposed the Conical function as a suitable alternative. The Conical function is formulated as following:

$$t(v) = \mathrm{t}_0[2 + \sqrt{\alpha^2\left(1-\frac{v}{c}\right)^2 + \beta^2} - \alpha\left(1-\frac{v}{c}\right) - \beta]$$

And,

$$\beta = \frac{2\alpha - 1}{2\alpha - 2}$$

Where,

t = Link travel time
t_0 = Free flow travel time
v = Traffic volume

c = Link Capacity
α = Parameter (any value larger than 1). Spiess (1990, p. 155)

Since the Conical function does not contain the same exponential characteristic of the BPR function (β parameter in BPR), therefore it is particularly suitable for relatively congested traffic condition. For an example, Community Planning Association of Southwest Idaho (2017, p. 48) used Conical VDF in their peak hour assignment model and they justified their decision by saying “since congestion is a more critical component for the peak hour, Conical VDF is used”. Kalaee (2010, p. 70) pointed out that a fundamental assumption in Conical function is that the estimated travel time at capacity is twice of the free-flow travel time.

Another advantage of Conical function is, as mentioned by Spiess (1990, p. 157), both BPR and Conical function have the same interpretation of the variables characterizing the traffic behavior, i.e., volume (v), and capacity (c). This makes the transition from BPR function to Conical function very convenient.

2.4.3. Akcelik Delay Function

Akcelik delay function has been developed based on an earlier delay function, known as Davidson’s function. Davidson (1966) originally proposed following function:

$$t = t_0 \; [1 + JX/(1 - X)]$$

where,
t =Average travel time per unit distance
t_o=Minimum (zero-flow) travel time per unit distance
J = Parameter
X = Degree of saturation (= V/C)

However, Golding (1977) discovered that there was inherent inconsistency in the original Davidson delay function which posed complexity to define the parameters and therefore caused difficulties in the calibration of the model. As a result, Davidson (1978) tried to define the delay parameter (J). However, Blunden (1978) pointed out that Davidson's modification still do not provide sufficient theoretical explanation for travel time on a continuously distributed traffic element

Finally, Akcelik (1991, pp. 52–54) formulated the time-dependent concept of Davidson's function using the coordinate transformation technique, which can be described as:

$$t = t_0 + 0.25\,T\;[(X - 1) + \sqrt{(X - 1)^2 + \frac{8JX}{cT}}]$$

Where,
t = average travel time per unit distance

t_o= minimum (zero-flow) travel time per unit distance
T = Assignment period

Subsequently, Transportation Research Board (2000) modified the Akcelik delay function in their Highway Capacity Manual (HCM 2000) into following formula:

$$R \ = \ R_0 + D_0 + 0.25T \ [\ (X-1) + \sqrt{(X-1)^2 + \frac{16JXL^2}{T^2}} \ ]$$

Where,
R = Link travel time
R_o = Link travel time at FFS
D_o= Zero-flow control delay at signalized intersection
L = link length

The zero-flow control delay for signalized intersections (D_o) on the link is computed using following equation:

$$\mathrm{D}_0 \ = \ \frac{N}{3600} * D_F * \frac{C}{2}\left(1 - \frac{g}{C}\right)^2$$

Where,
N = Number of signals on link
g = Average effective green time for signals on link
C = Average cycle length for all signals on link
D_F = Adjustment factor to compute zero-flow control delay (0.9 for uncoordinated traffic-actuated signals, 1.0 for uncoordinated fixed-time signals, 1.2 for coordinated signals with unfavorable progression, 0.90 for coordinated signals with favorable progression, and 0.60 for coordinated signals with highly favorable progression).

The calibration parameter J is defined in such a way that the travel time equation will predict the mean speed of traffic when demand is equal to capacity. Thereby, substituting X = 1 in the link travel time equation, the solution J yields following Equation:

$J = \frac{(R_c - R_0)^2}{L^2}$ Transportation Research Board (2000, pp. 30_6–30_7)

However, Caliper Corporation (2017) simplified the HCM 2000 formula in order to make the UE solution computationally more efficient:

$R = R_0 + D_0 + 0.25 \ T \ [(X-1)^2 + \frac{16JXL^2}{T^2}]$ Caliper Corporation (2017, p. 199)

2.4.4. Logit-based Volume-Delay Function

The Israel Institute of Transportation Planning and Research developed a logit-based function that takes both link and intersection delay into account. However, Jeihani et al. (2006, p. 4) noted that the intersection delay of Logit-based function is not volume-based. The total delay on a link is calculated as following:

$$D = D_l + I_l$$

Where,
D_l = Link delay
I_l = Intersection delay

And link delay (D_l):

$$D_l = t_0\, a_1 \left[\frac{1}{1 - \frac{a_2}{1 + e^{(a_3 - a_4 \frac{v}{c})}}}\right]$$

t_0 = Free-flow travel time
v = Traffic flow
c = Link capacity
a_1, a_2, a_3, a_4 = Parameters

And, node delay (I_l):

$$I_l = d_0\, b_1 \frac{b_2}{[1 + e^{(b_3 - b_4 \frac{v}{X})}]}$$

d_0 = Free-flow travel time of intersection
X = Intersection capacity which can be calculated as a function of the link capacity and the expected percentage of green light for a signalized intersection
b_1, b_2, b_3, b_4 = Parameters   Caliper Corporation (2017, pp. 198–199)

Lu et al. (2010, p. 1993) mentioned an advantage of logit-based delay function over the HCM 2000 version of Akcelik delay function. The logit-based function requires less intersection data as inputs than Akcelik function.

2.5. Comparative Analysis on VDF

Cheah et al. (1992) investigated BPR, Conical and some other alternative delay functions using EMME/2. In addition to the base scenario, the tests were also conducted on a simulated over-capacity scenario with 150% of the base year demand. The performance of Conical delay function was compared to the one of BPR in terms of link volume, travel time and speed of convergence.

Dowling et al. (1998) compared different forms of BPR and Akçelik delay function with respect to various indicators. First, they investigated two indicators, which were speed vs. V/C ratio and travel time vs. V/C ratio. In these test, the real-world data collected from a freeway and a signalized arterial in Southern California was used for V/C ratio<1; however, for V/C ratio>1, they used FREQ and TRANSYT-7F simulation result. Subsequently, other indicators such as vehicle kilometers traveled (VKT) vs. speed, running times and speed of convergence were also tested for two different models (MINUTP based San Francisco Metropolitan Transportation Commission model and EMME/2 based Contra Costa Transportation Authority Tri-Valley model).

Singh and Dowling (1999) conducted a detailed comparative study between the BPR and Akcelik delay function for the highway network in San Francisco Bay area, United States. The VDFs are compared in terms of computing times, speed vs. V/C ratio, travel time vs. V/C ratio, vehicle-miles traveled (VMT) vs. V/C ratio, vehicle-hours traveled (VHT) vs. V/C ratio, percentage of vehicle-miles vs. speed, speed by facility types, and vehicle-miles by facility type.

Lee et al. (2009) tested BPR, Conical and Akcelik delay function using real-world observation data for each road facility class in Virginia, United States. They compared the speed vs. density relationship of those VDF, checked the “goodness of fit” using RMSE and R-squared as well as calibrated the associated VDF parameters.

Kalaee (2010) compared the travel time estimates produced by BPR, Conical and Akçelik delay function with the empirical data from the Portland Oregon Regional Transportation Archive. All VDFs were tested using bias, RMSE, Mean Absolute Error (MAE) and fitted to the observation data iteratively in order to calibrate the parameters.

Klieman et al. (2011) plotted the speed vs. V/C ratio performance of BPR, Conical, Akcelik and Logit-based delay function for each facility type in Maricopa County, United States. Then the “goodness to fit” of those four VDFs were tested against the empirical data using Root Mean Square Percent Error (RMSPE).

Mtoi and Moses (2014) did a comparative analysis among BPR, Conical, Akcelik and a modified form of Davidson’s (1966) delay function based on Orlando Urban Area Transportation Study data. Their research was focused on three elements. First, a comparative analysis of speed vs. congestion level (V/C) is performed along with the actual field data to capture the functional characteristics of VDF. Consecutively, the sensitivity of speed with respect to the congestion level was studied through the slope of the first curve. And finally, the performance of each VDF was analyzed by facility type and their respective RMSPE values were calculated in order to evaluate the facility specific performance of each VDF.

3. Algorithm of TransCAD DTA

TransCAD DTA algorithm is developed as an extension of the static UE, such that the assignment period is divided into finer temporal granularity and at each temporal interval, no user can reduce his/her experienced travel time by unilaterally changing the route. The experienced travel time is calculated by VDF, based on which the traffic flow is distributed throughout the network. This process is run iteratively until the DUE reaches to a converged solution.

The DTA procedure in TransCAD is influenced from the approach developed by Janson (1991b) and Janson and Robles (1995), which was later modified and extended by Caliper Corporation (2017). The DTA solution closely satisfies a temporal extension of the first principle of Wardrop (1952), i.e. all used paths between a given origin-destination (OD) pair for the same departure time have the same and minimum experienced travel time. TransCAD addressed the DTA problem as a mathematical programming-based problem and the DUE is formulated as a constrained optimization problem. The UE procedure, in general, is based on a solution method that is more rapidly convergent than the conventional Frank-Wolfe (FW) method proposed by Frank and Wolfe (1956). This method is known as conjugate direction FW (Conjugate UE) method that was originally introduced by Daneva and Lindberg (2003) and later explained in detail by Mitradjieva and Lindberg (2013). Caliper Corporation (2017) noted that this method yields quicker convergence than the original FW method and usually reaches to a lower relative gap in the same amount of computing time. They also suggested conducting some experiments in order to specify the ideal value for conjugates, because the higher conjugates can be more efficient.

The assignment period is divided into multiple time-intervals, each with a constant length. The time-interval represents the discretization of O-D demand. After defining all intermediate and output variables and carrying out all calculations, the DTA model produces the model outputs for each time-interval. Although, the number of time-intervals are not restricted to a certain limit, but there is a trade-off between model accuracy and computational efficiency. A shorter time-interval can offer more detailed information about the traffic conditions, however it leads to longer computational times and greater memory requirements. The TransCAD DTA algorithm recognizes two types of time-interval, the departure interval and the total interval. Both time-intervals have the same length, but the number of total interval is either equal or greater than the number of departure interval. Each departure interval corresponds to an OD matrix. The vehicle trips depart from each O-D matrix in each departure time-interval. On the other hand, the total intervals refer to all intervals during model assignment period. If the number of total interval is greater than the number of departure interval, then it means that there are additional intervals after the departure intervals which are to ensure that all trips which are already introduced in the network can be completed within the assignment period.

In order to extend static UE to DUE, a common heuristic would be to apply static UE sequentially to all time-intervals, while carrying flows from the previous period to current period. However, with this approach, the calculation of dynamic path travel time has a couple of flaws. First, in a static UE for a specific time interval, the evaluation of the trip travel time is based on link travel time (instantaneous travel time) at this time interval, yet the time span of the trip could include the next time-interval or even the later time-interval. Thus, the correct calculation should include the composition of link travel time based upon the arrival times at the links. Secondly, the separation of dynamic OD trips into different static traffic assignment models ignores the interaction among OD trips from all time periods. The travel cost experienced by a traveler who departs at a given time period is potentially affected by other trips departing at earlier and later time periods. Therefore, all dynamic OD trips should be assigned in a single equilibrium model, which is precisely what equilibrium DTA model performs.

The DUE solution algorithm contains two levels of iterative loops, an outer (Node-Time-Arrival) loop and an inner (assignment) loop. The outer process constantly solves for a three-dimensional matrix that governs the dynamic propagation of OD flows in the network. And the inner process solves for an equilibrium assignment internally for a given node-time-arrival matrix. The assignment loop is a process similar to a FW static assignment. On the other hand, the outer loop updates a three-dimensional array of variables called Node-Time-Arrival (NTA) which specify how flows spread out temporally. This NTA can be roughly viewed as a temporal extension of the link-path incidence matrix of static assignment problem. The algorithm of the TransCAD DTA model can be visually illustrated as *Figure 1*.

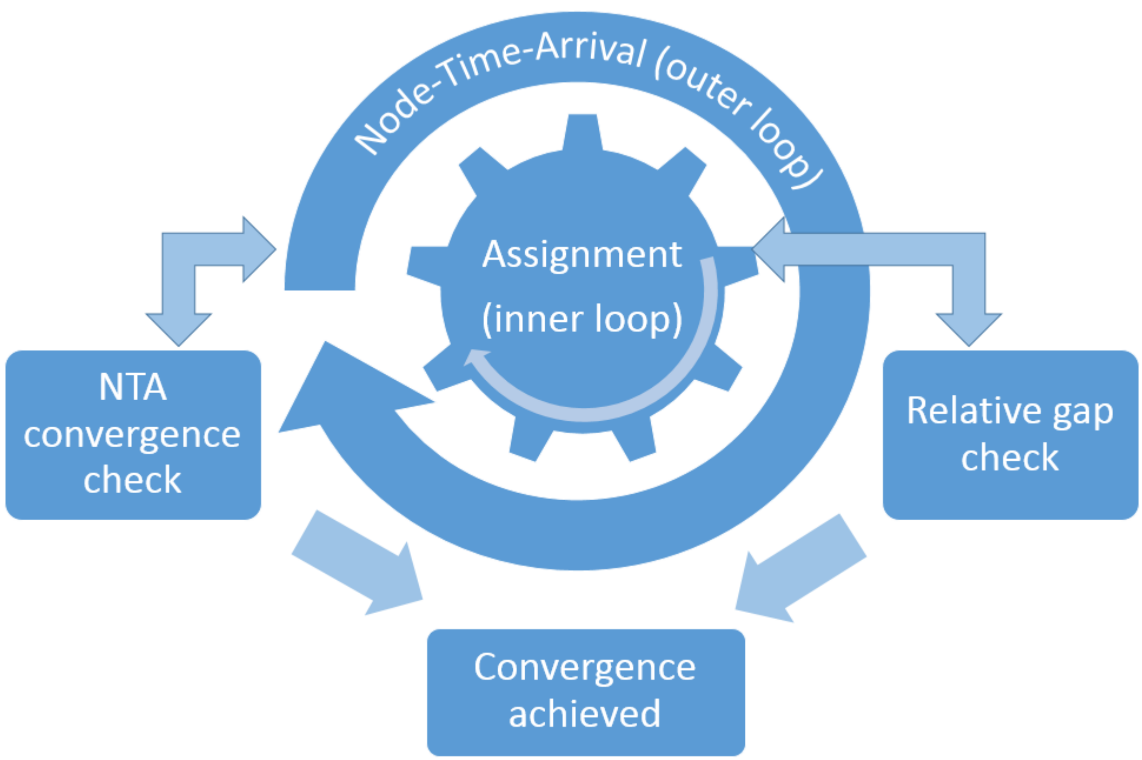

Figure 1: Algorithm of TransCAD DTA model

Mathematical programming-based DTA has a long-standing issue regarding the trade-off between mathematical tractability and modeling ability, i.e., inconvenient constraints vs. traffic realism. To list the limitations of mathematical programming-based DTA, Peeta and Ziliaskopoulos (2001, p. 237) as well as Szeto and Wong (2012, pp. 12–13) identified the “holding-back of traffic”, spill-back and FIFO (First-In-First-Out) as three major challenges. However, Caliper Corporation (2017, p. 271) claimed that the TransCAD DTA procedure extends the original Janson's mathematical programming-based DTA method in several aspects with improved algorithm and convergence that yields more consistent calculations. As a result, FIFO, holding-back and spill-back issues are resolved. In a personal email correspondence, Balakrishna, Caliper Corporation (3/12/2018) unofficially explained that there are heuristics to “hold-back” flows on some links to allow traffic flows to be realistic. These calculations also allow TransCAD to introduce spillbacks and queues.

The quality of the DUE solution flow vector depends on the level of convergence of the iterative algorithm. Owing to its convexity in objective function and constraint sets, the mathematical programming-based DUE problem used in TransCAD offers an unique optimal assignment solution. While reaching towards the optimal assignment solution, the link flow difference between two consecutive iterations diminishes. Besides, the diminishing link flow difference is termed as the “proxy measure of the equilibrium condition” by the U.S. Department of Transportation, Federal Highway Administration (2012, pp. 2–6) and they also recommended it to use as the convergence criterion.

A well designed convergence criterion is very important in order to achieve a stable and unique solution. TransCAD DTA algorithm uses different convergence criteria for its two levels of iterations. For the convergence of outer NTA iteration loop, the relative change of NTA matrix in consecutive iterations is defined as the convergence criterion. On the other hand, the relative gap is set as the criterion for inner assignment iteration loop.

The DTA algorithm will stop if either of the following two actions is executed:

- Two conditions are fulfilled simultaneously. Firstly, the relative change of the NTA matrix since the last iteration is less than the NTA convergence criterion. And secondly, within the current outer iteration (node arrival iteration), the relative change of two subsequent assignment iterations is also less than the relative gap. Therefore, both node arrival and assignment convergence are achieved.
- The number of total iterations ("NTA Iterations" times “Assignment Iterations”) is reached.

When the first action is realized before the second condition, both iterative loops are converged and the DUE is reached to a NTA matrix that is consistent with actual link travel times.

4. Data Overview and Model Development

4.1. Study Area and Traffic Analysis Zone (TAZ)

Traffic Analysis Zone (TAZ) represents a spatial unit that provides the socio-economic data required for the calculation of the trip demand. The traffic demand data and associated TAZ system used in this assignment model is originally based on the data provided by the *Professorship of Modeling Spatial Mobility* research group of TU Munich. The *Professorship* developed an integrated land use and transportation model for greater Munich Metropolitan area. Their TAZ system was created by using a gradual zoning algorithm by Moeckel and Donnelly (2015), which automatically defines optimal-sized zones based on raster cells of socio-economic data. The original TAZ system contained a total of 4953 zones covering the greater Munich Metropolitan region, including five cities: Augsburg, Ingolstadt, Landshut, Munich and Rosenheim. The zone sizes in the urban areas were smaller in comparison with larger zones in the rural areas. However, the study area for this research is focused on central Munich area as shown in *Figure 2.*

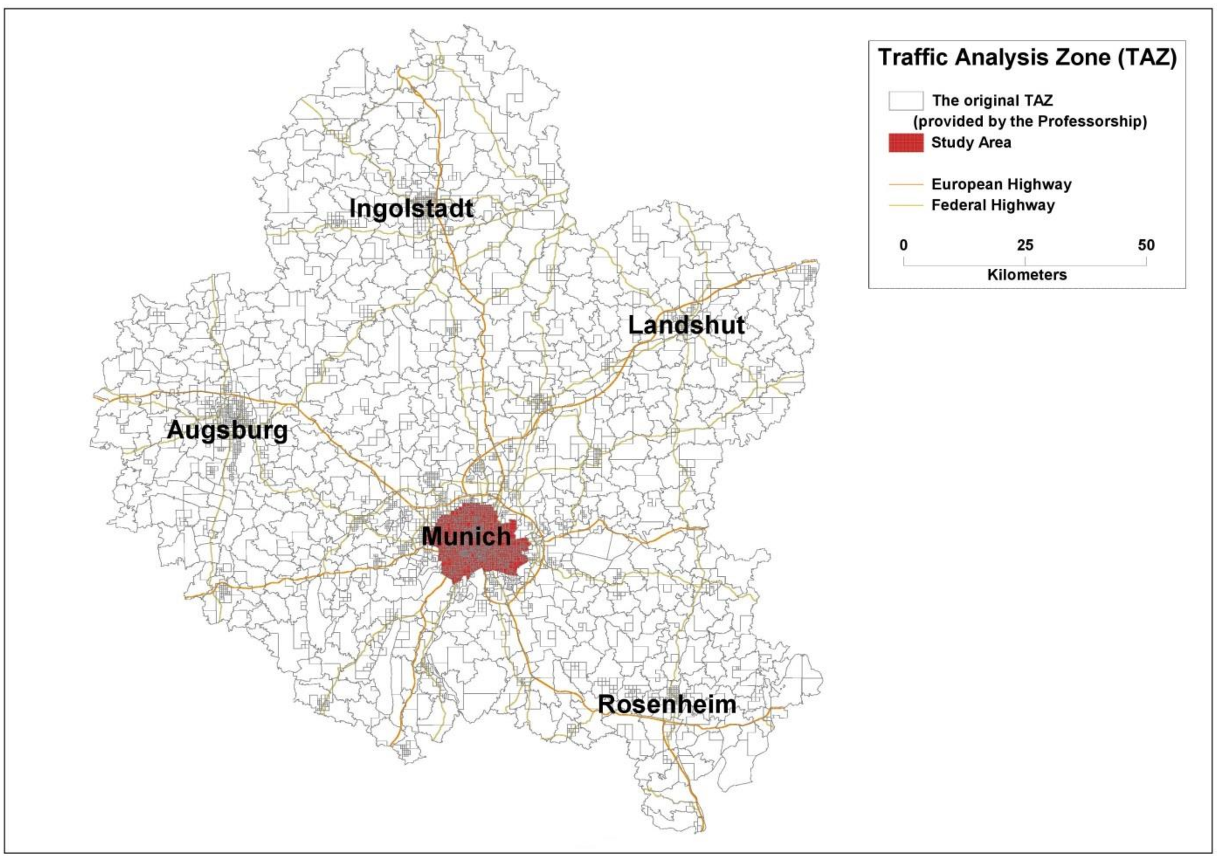

Figure 2: Original TAZ and Study Area

The study area is narrowed down to central Munich due to following reasons:

- The demand data of the Munich city is understood to have a better quality than the one of the surrounding rural area.
- The smaller size of zones in the urban area will produce more detailed traffic flow at the lower facility of the road network.
- The traffic around the boundary of the Munich city administrative territory is affected by the federal highway ring road (A99), which encircled the Munich city.

Depending on different possible major entries into the study area, all zones from the original TAZ system that left outside the study area are aggregated into 10 external zones in order to calculate the external traffic demand. *Figure 3* demonstrates how the zones around the Munich regional area are aggregated into 10 external zones.

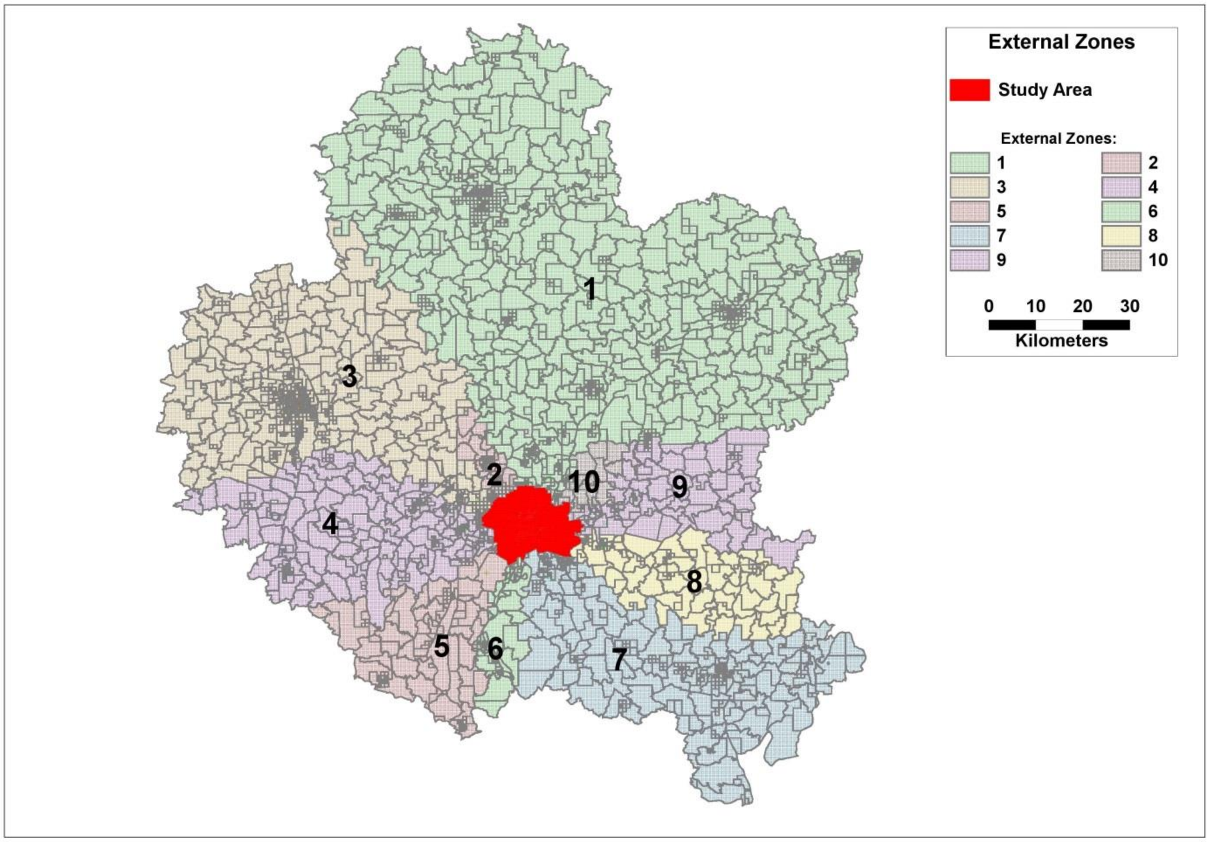

Figure 3: External Zones

A total of 1752 zones fit within the study area from the original TAZ system. However, after few test runs, it appears that the zone sizes are too small and the number of zones are too high for the study area such that it causes an unnecessary computational burden without adding much value to the assignment flow. Therefore, the original TAZ system is modified and the zones are aggregated and merged together in such a way that the center of the study area contains the smallest sized zones only while the zone size gets bigger as the distance from the center increases. After the modification, the final TAZ structure contains 750 zones inside the study area. The *Figure 4* shows a side-by-side comparison between the original TAZ system within the study area and final the modified TAZ structure used in this research.

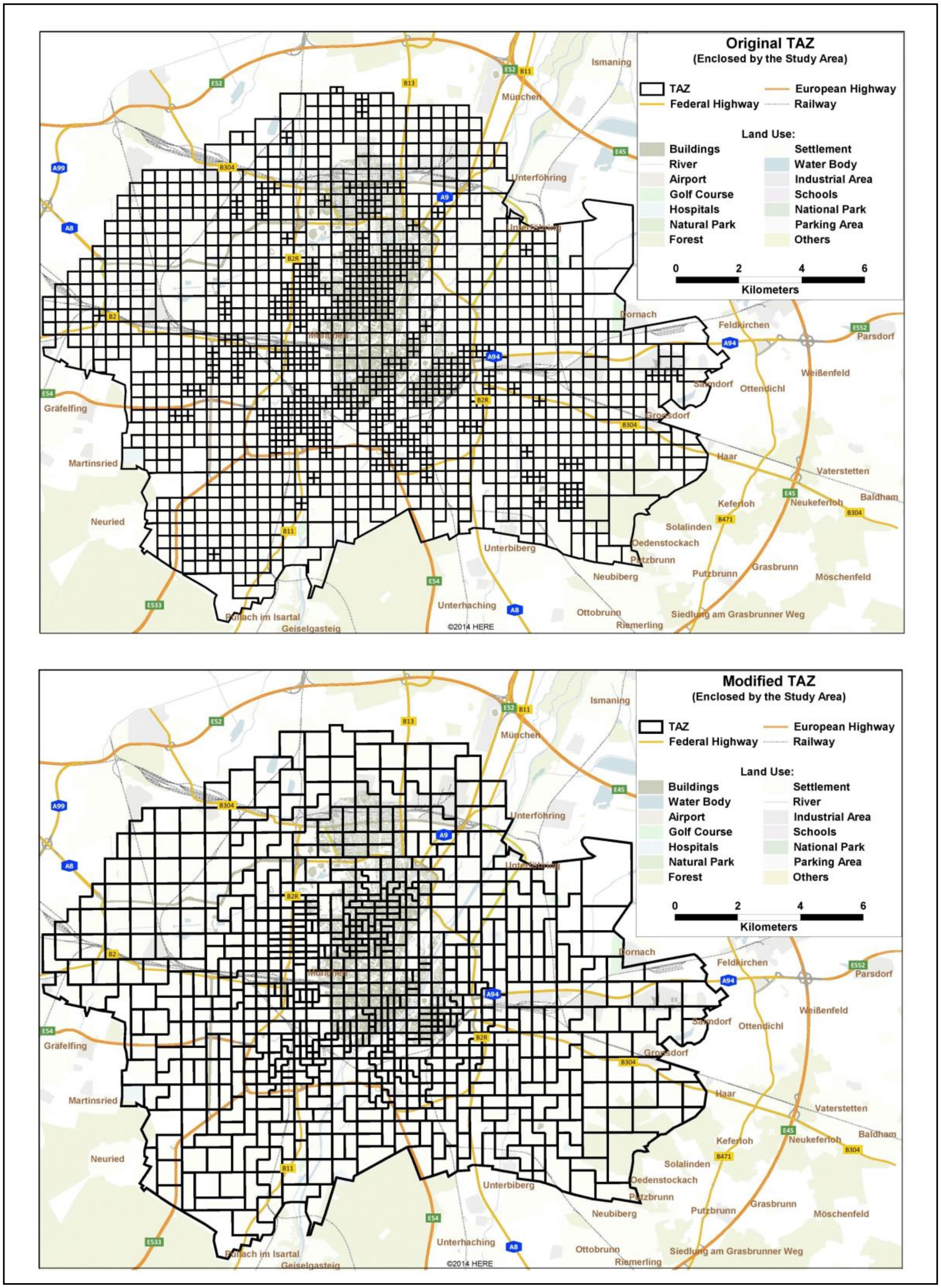

Figure 4: Modification of TAZ

4.2. Time-interval

The base model scenario for this research is considered to be the year of 2011, so that it is consistent with the traffic demand data of the greater Munich Metropolitan model which was also developed for the same year. The distribution of traffic demand across the time horizon for an average working day in 2011 is obtained from the publicly available automatic traffic count data of *Bundesanstalt für Straßenwesen (BASt)*. *Figure 5* shows the locations of the traffic count stations whose data were used in this research.

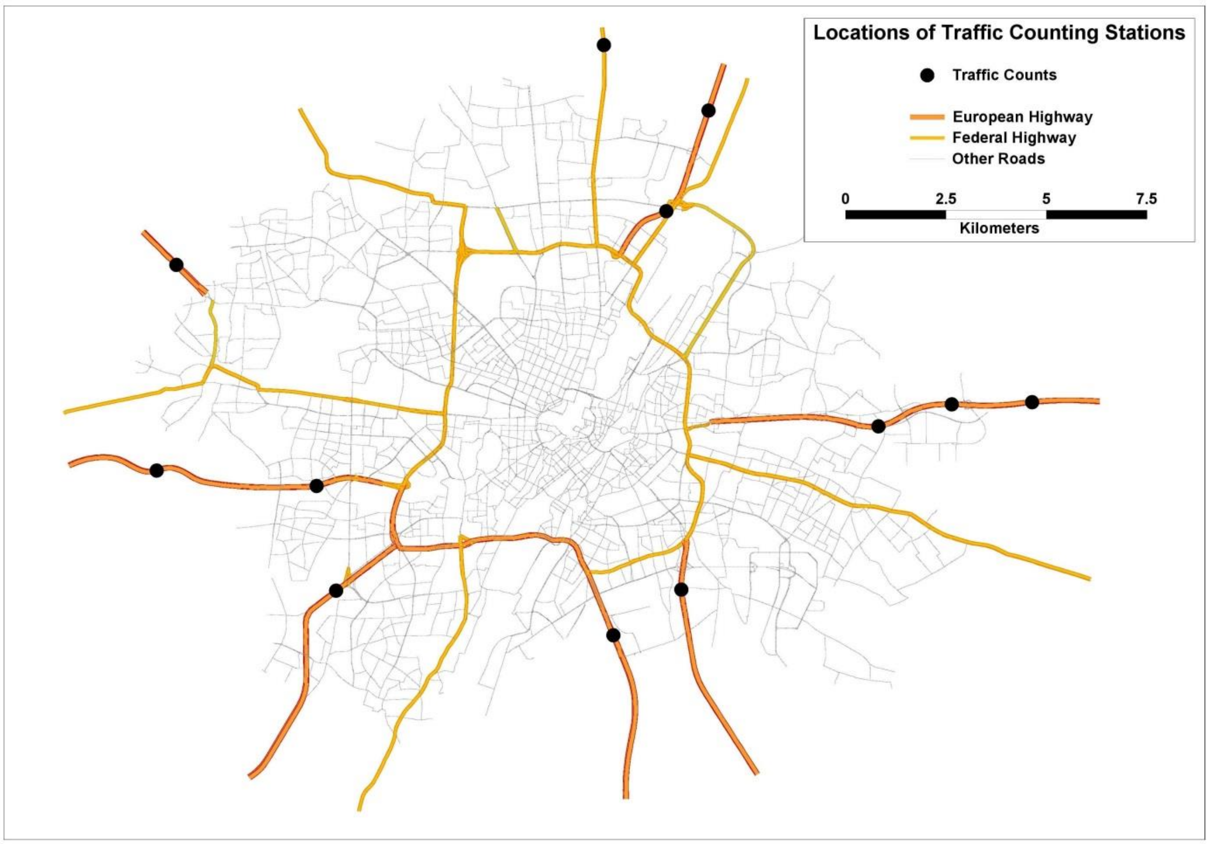

Figure 5: Locations of Traffic Counting Stations

The traffic count data of these 12 Stations are analyzed using statistical analysis software R. The hourly traffic data of each working day of the whole year of 2011 is analyzed and an average hourly traffic demand for a full day is obtained. The distribution of the average hourly traffic demand for a normal working day is presented in *Figure 6*.

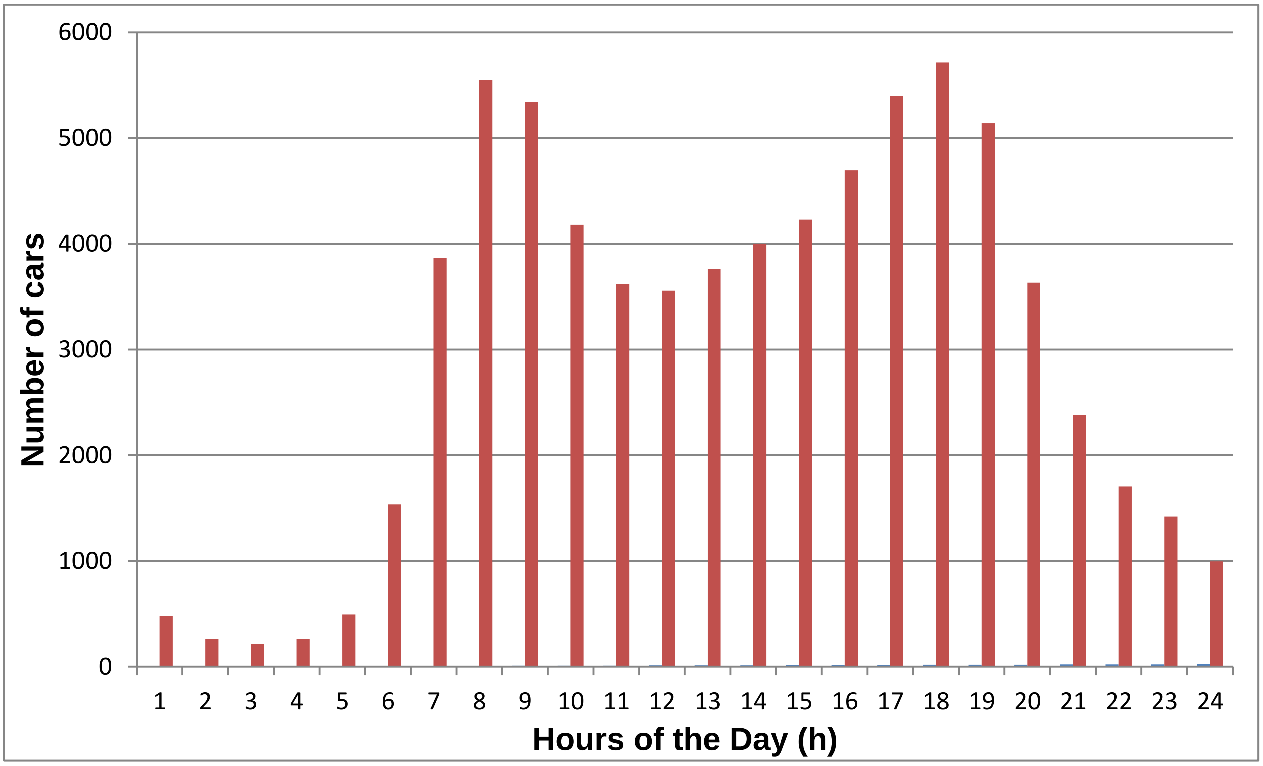

Figure 6: Distribution of Daily Traffic Demand

The *Figure 6* shows two peaks in the traffic distribution, one at the morning (7:00 – 8:00) and another at the afternoon (17:00 – 18:00). As a result, it is decided to run the dynamic assignment model twice, at first for morning peak-hour and then again for afternoon peak-hour. Each peak hour period is subjected to a total assignment window of 90 minutes. The initial 30 minutes will be allocated for filling the empty-network and the final 60 minutes are reserved for actual analysis purpose. Each peak-hour assignment period is comprised of 9 assignment intervals, with each interval being a slice of 10-minutes duration. Therefore, two peak-hour assignment periods (morning and afternoon) contain a total of 18 assignment intervals, of which 12 intervals are for the actual analysis.

4.3. Traffic Demand (OD Matrix)

According to Llorca et al. (2017), the greater Munich Metropolitan area model is a mesoscopic model, whose assignment is to be performed on MATSim using a trip demand table. This trip demand table is comprised of sequential individual trips estimated for the period of a whole working day. Each individual trip contains an origin zone ID, a destination zone ID and a trip departure time.

The TransCAD based macroscopic model, however, is unable to directly utilize the trip demand table of the MATSim based mesoscopic model. The model used in this research requires the traffic demand input data as OD matrix. Therefore, the trip demand table needs to be aggregated into OD matrix. However, before aggregation of individual trips, it has to be ensured that the origin and destination zones' IDs from the original trip table is consistent with the modified TAZ system. It is necessary because of the original TAZ system was modified, i.e., many zones inside the study area were merged along with the 10 external zones as it is illustrated in *Figure 4*, and Figure 3 respectively. Afterwards, all individual trips are aggregated in order to produce different OD matrices based on the trip departure time. Each OD matrix is accounted for an assignment interval (10-minutes). At the end, a total of 18 OD matrices (9 matrices for morning peak-hour and another 9 for afternoon peak-hour) are created.

4.4. Network

A network is a special data structure that stores connectivity, link, and node characteristics of the transportation systems and facilities. Networks are defined, derived, and used in conjunction with a line layer and its associated node layer. The network used in this research, is originally based on the *HERE Map data 2014*. However, the HERE network within the study area was extensively detailed with each and every local/residential road and so many unpaved pedestrian pathways. Thus, the total of 45160 links and 31967 nodes of the original HERE network are deemed as "too detailed" for a macroscopic model such that it would pose an unnecessary computational challenge. Therefore, the original HERE network is modified by filtering out some of the lower level road facilities (mostly, local/residential streets and pedestrian pathways) from the network. As a result, the total number of links and nodes in the modified road network are reduced down to 15542 links and 13233 nodes (excluding connectors and centroids) respectively. After conducting few test assignment runs, it seems that the modified network shows incredible improvement in computational efficiency compared to the original HERE network, while the assignment results are still almost the same.

The link properties (such as speed, capacity etc.) are dependent on respective link category. The link categorization is accomplished based on the free-flow speed. The HERE network data originally came with default free-flow speed for each link, which are later slightly adjusted, so that the adjusted values precisely fit within the free-flow speed range of different road types defined in the Transportation Research Board (2000). At the end, the links are categorized into 9 facility types, in line with the road classification of Transportation Research Board (2000). Regarding the link capacity, Cambridge Systematics et al. (2012, p. 20) referred to the Transportation Research Board (2000) as the "definitive reference for defining highway capacity". Moreover, Cambridge Systematics et al. (2012, p. 74) and Horowitz (1991, pp. 11–12) recommended Level of Service (LOS) E as the link capacity for macroscopic transport models, since LOS E is equivalent to ultimate capacity. Therefore, the LOS E capacity from Transportation Research Board (2000, pp. 10_10, 10_35, 12_11 & 13_13) was defined as the link capacity for each link category. *Table 1* shows the link classifications, number of links in each category and their respective properties.

Table 1: Network Properties

Facility Type	Number of Lanes per Direction	Free Flow Speed (km/h)	Hourly Capacity (veh/h/ln)	Number of Links	Total Length (km)
Freeway	2	110	2055	206	45.7
	3		2067		
	4		2078		
	5		2090		
Highway class-I	2	100	2030	148	23.1
	3		2033		
	4		2035		
Highway class-II	2	80	1760	267	36.8
	3		1763		
	4		1765		
Urban Street class-I	1	70	1110	1261	115.2
	2		1060		
	3		1013		
	4		1015		
Urban Street class-II	1	60	860	1307	117.7
	2		825		
	3		790		
	4		798		
Urban Street class-III	1	50	840	4163	299.1
	2		805		
	3		770		
	4		770		
Urban Street class-IV	1	40	790	3360	277.3
	2		760		
	3		727		
	4		725		
Minor Urban Street	1	25	350	4576	387.2
	2		260		
U-turn	1	10	200	254	3.8
	2		175		
	3		150		
Total				**15542**	**1306.0**

As presented in *Table 1*, 15542 links are divided into 9 categories, ran across a network with a total length of 1306 kilometers. A distribution of these link categories in the network is displayed in *Figure 7*.

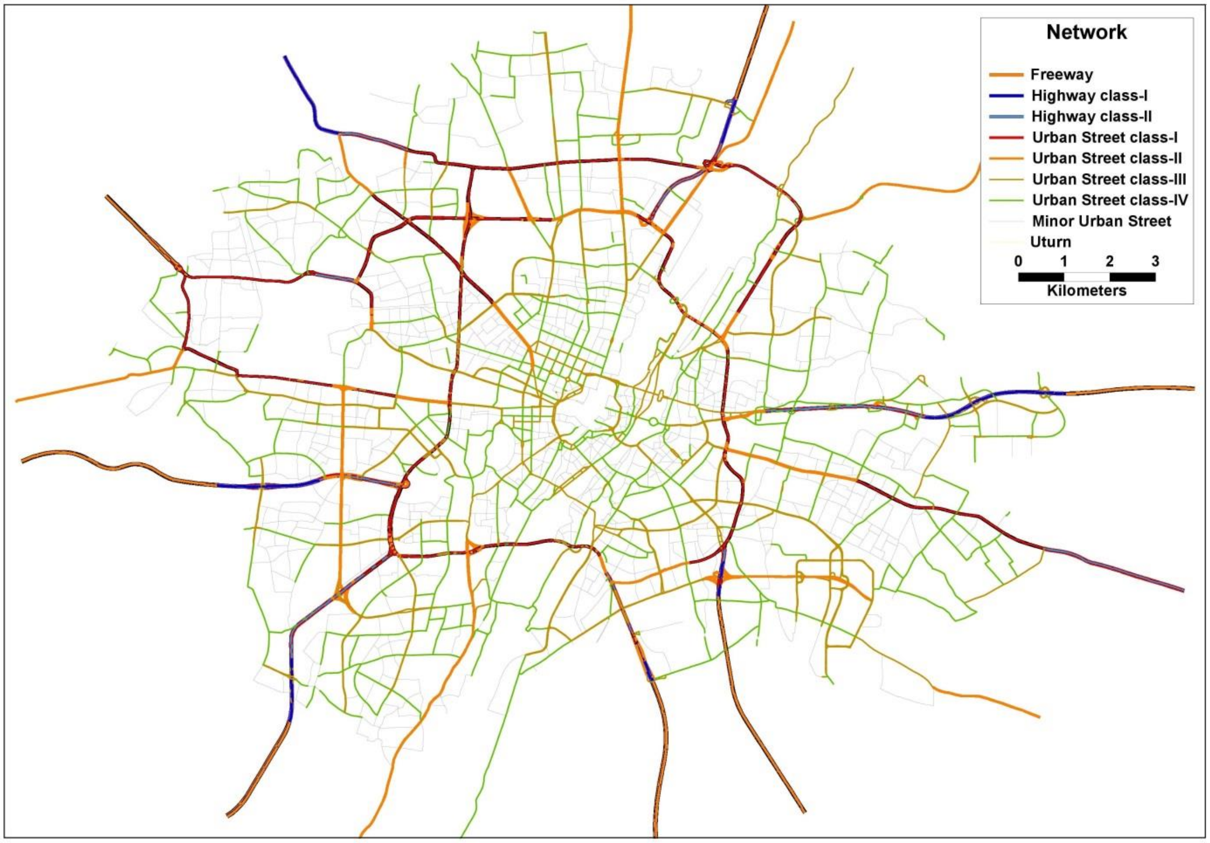

Figure 7: Overview of Link Categories in the Network

To above physical road network, some hypothetical nodes and links are also added, which are known as centroids and connectors respectively. Ortúzar and Willumsen (2011, p. 130) defined the centroid as an imaginary single point in which all properties of a zone is concentrated and a connector is referred to a theoretical line connecting the centroid to the nearby physical road network. One centroid-node is pinned at the center of gravity of each TAZ. Thus, among a total of 760 centroids, 750 centroids represent internal zones and the remaining 10 are for external zones. Likewise, 1500 connectors for the internal zones (two connectors for each internal-centroid) are added to the network, plus 27 additional connectors connect the external-centroids to the road network. Therefore, the total number of connectors becomes 1527.

In relation to the properties of the connectors, Transportation Research Board (2000, 30.5) stated that the connectors are not subject to any regular facility type and typically have a fixed free-flow speed and near-infinite capacity. Therefore, high capacity and free-flow speed values are used in this model, in order to avoid any possibility of congestion inside the connectors. *Figure 8* illustrates the complete network, combining the centroids and connectors along with the physical road network. This figure also shows the position of centroids with respect to the zones and the alignment of connectors are displayed with a zoomed in view.

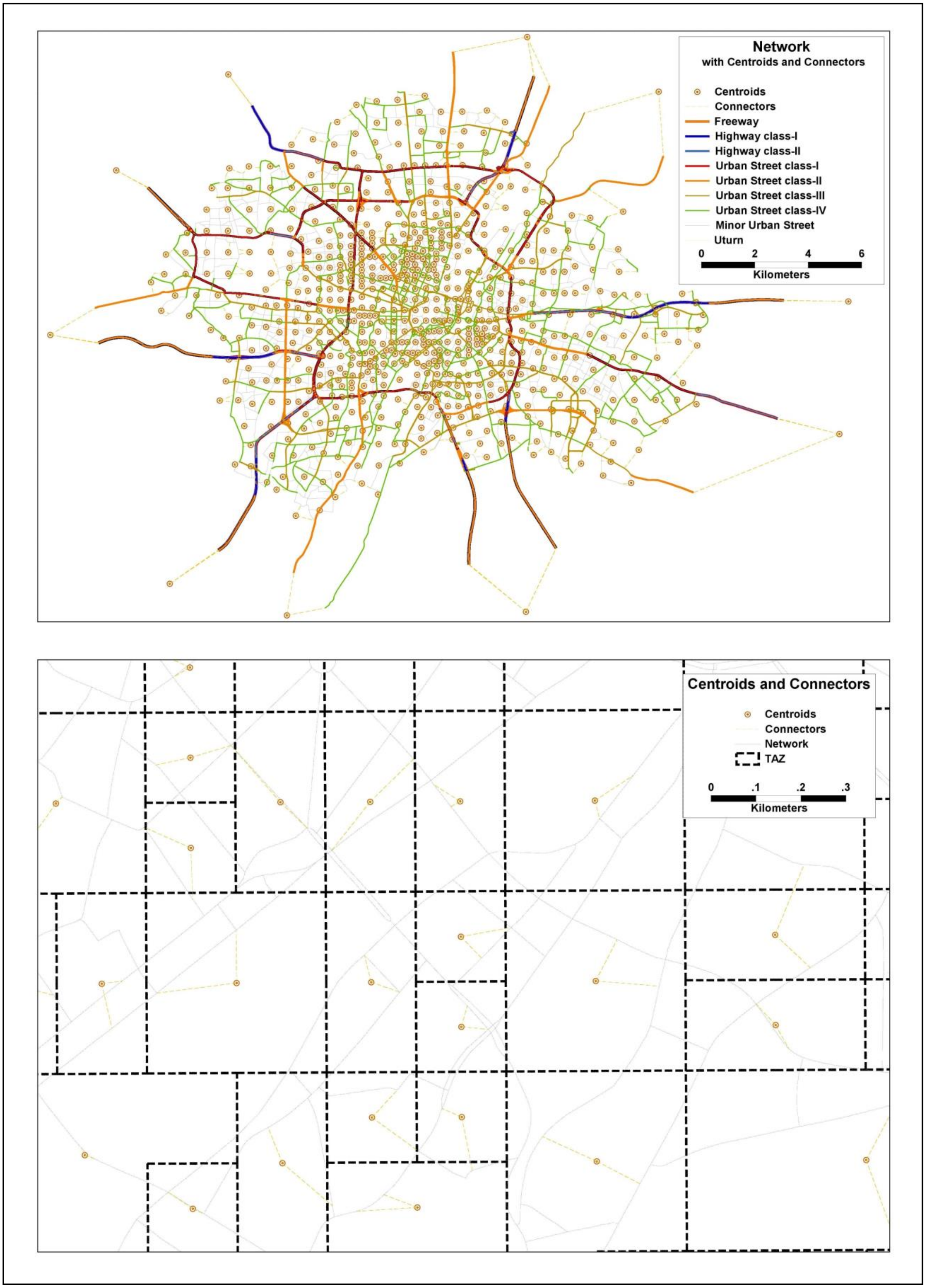

Figure 8: Centroids and Connectors

4.5.Intersection data

Akcelik and logit-based VDF take intersection delay into consideration to calculate the link travel time of any link that approaches towards a signalized intersection. The Akcelik function requires the exact value of effective green time (g) and cycle time (C) in order to calculate the intersection delay. On the other hand, the logit-based function needs only a percentage of effective green time (g) for a signalized intersection.

Adding very precise intersection level details into a macroscopic model is, however, quite a challenging task. More importantly, accurate data for so many intersections in such a large network are most often unavailable, as it is the case in this study. Therefore, an approximation of the intersection data is estimated using the following approach:

- Only the locations intersected by the mid-level road facility, i.e., Urban Street class-I, II, III and IV (a range of 70-40 km/h FFS) are considered to be the signalized intersections. The remaining intersecting points crossed by higher or lower level road facilities were assumed to be unsignalized. As a result, a total of 1708 points are selected for signalized intersections. *Figure 9* exhibits an illustration of all the assumed locations for signalized intersection over the whole network.

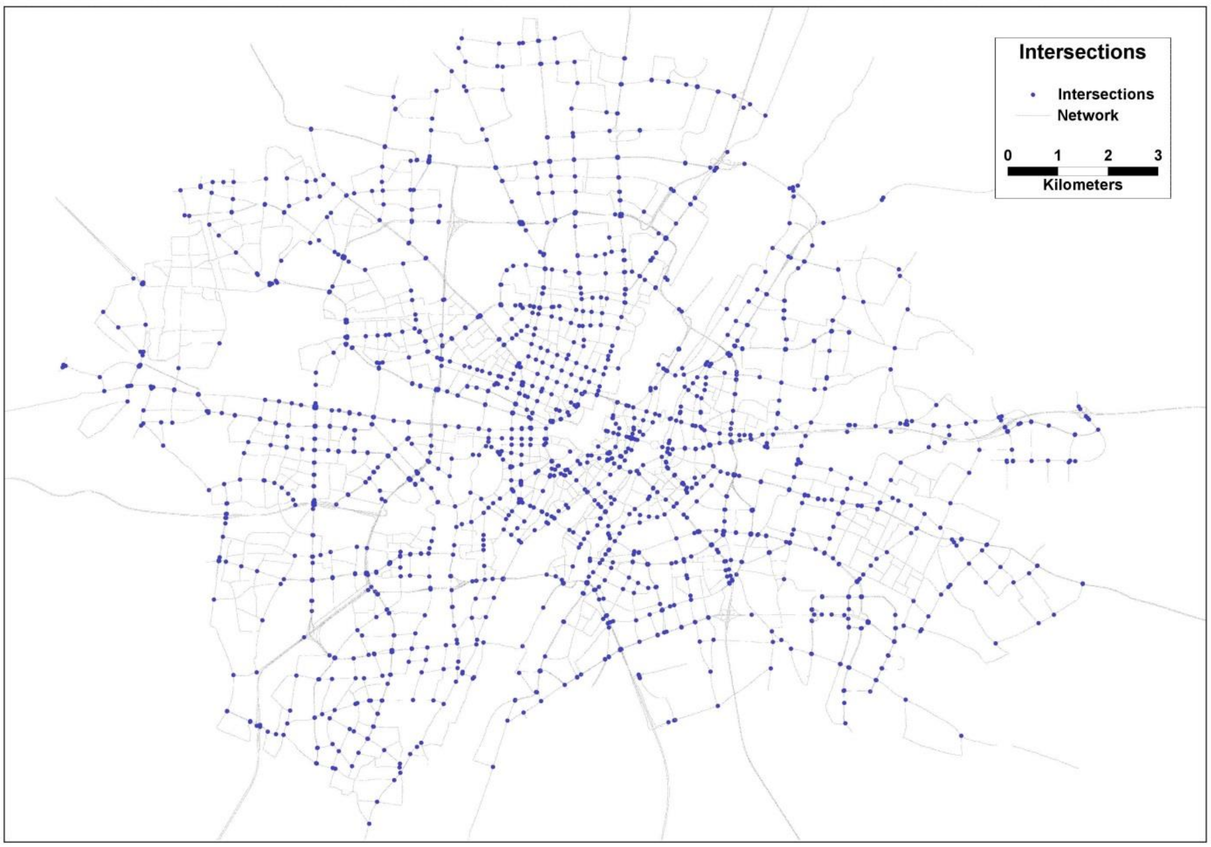

Figure 9: Intersections in the Network

- The number of possible turning movements in each of those 1708 signalized intersections is recorded.
- According to the method explained in Transportation Research Board (2000, p. 16_95), based on the number of turning movements, the number of signal phases are obtained for each signalized intersection.
- From the field observation, approximate values for cycle time and green time are defined with respect to the number of signal phases.
- Transportation Research Board (2000, p. 10_22) listed how much green time is usually lost in each cycle depending on the number of signal phases. These lost time per cycle values are taken.
- The effective green time per cycle is then calculated by subtracting the lost time from the original green time.

Accordingly, the intersection related variables such as the percentage of effective green time (for Logit-based VDF) and Zero-flow Control Delay (for Akcelik VDF) are calculated from the ratio of effective green time and cycle time. All the above mentioned values are presented in *Table 2*.

Table 2: Calculation of Intersection Delay

Turning Movements	**Number of Phase**	**Cycle Time (sec)**	**Green Time (sec)**	**Loss Time per Cycle (sec)**	**Effective Green Time (sec)**	**Ratio of Effective Green Time & Cycle Time**	**Zero-flow Control Delay (min)**
3	2	60	30	8	26	0.43	0.16
4	2	60	30	8	26	0.43	0.16
5	2	60	30	8	26	0.43	0.16
6	2	60	30	8	26	0.43	0.16
7	3	75	25	12	21	0.28	0.32
8	3	75	25	12	21	0.28	0.32
9	3	75	25	12	21	0.28	0.32
10	3	75	25	12	21	0.28	0.32
11	4	80	20	16	16	0.20	0.43
12	4	80	20	16	16	0.20	0.43
13	4	80	20	16	16	0.20	0.43

Finally, 4151 links are found in the whole network, which are subject to intersection delay. A zoomed-in illustration of these special links is provided in *Figure 10*.

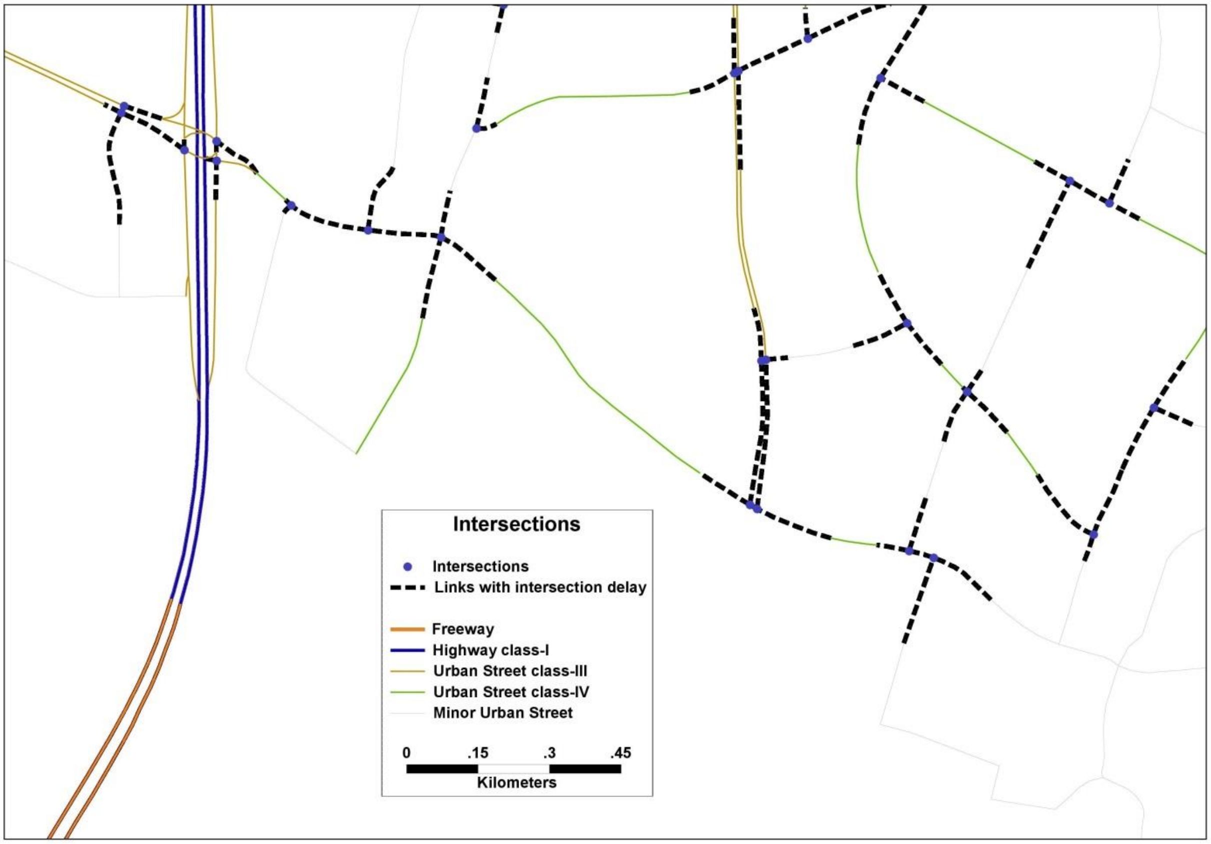

Figure 10: Distribution of Intersection Delay

4.6. VDF Parameters

All volume-delay functions contain certain variables whose values can be manipulated within a specific range; these variables are known as VDF parameters. The VDF parameters provide fair amount of control over the function result. Both Conical and Akcelik functions have only one parameter each, on the other hand, BPR and logit-based functions have 2 and 8 parameters respectively. The parameter values used in this research are collected from different credible sources, which are described below:

4.6.1. Bureau of Public Roads (BPR)

The two parameters used in BPR function are widely known as α (alpha) and β (beta). Kalaee (2010, p. 70) provided an explanation on the meaning of these parameters. The α parameter settles the ratio of travel time at free-flow to that at capacity. On the other hand, β defines how rapidly the travel time increases from the free-flow travel time. Since β being the power over V/C ratio in the BPR function, the smaller values of β makes estimated travel time more sensitive to V/C ratio.

Horowitz (1991, p. 12) recommended two sets of values for BPR parameters, one set for freeway and another for multilane highway facilities. Cambridge Systematics et al. (2012, p. 75) also referred to the same values recommended by Horowitz, however they noted that the term “freeway” and “multilane highway” were not properly defined. In the framework of the network used in this research, the term “freeway” is adopted for the top 3 road facility levels (freeway, highway class-I and II) and the remaining 6 lower link categories are considered to be appropriate for the “multilane highway” parameter set. Both sets of values are extended by applying non-linear regression in order to obtain parameter values for all 9 road categories. *Table 3* shows the final values for α and β parameters obtained after non-linear regression:

Table 3: BPR Parameter Values

Facility Type	Free Flow Speed (km/h)	Alpha (α)	Beta (β)	Approach
Freeway	110	2.30	71.00	Non-linear regression of "freeway" set
Highway class-I	100	1.83	43.02	
Highway class-II	80	1.17	15.80	
Urban Street class-I	70	1.00	5.40	Original values of “multilane highway” set
Urban Street class-II	60	0.83	2.70	
Urban Street class-III	50	0.71	2.10	
Urban Street class-IV	40	0.59	1.96	Non-linear regression of "multilane highway" set
Minor Urban Street	25	0.46	1.34	
U-turn	10	0.36	1.01	

The BPR parameter values contained in *Table 3* are illustrated graphically with respect to free flow speed in *Figure 11*.

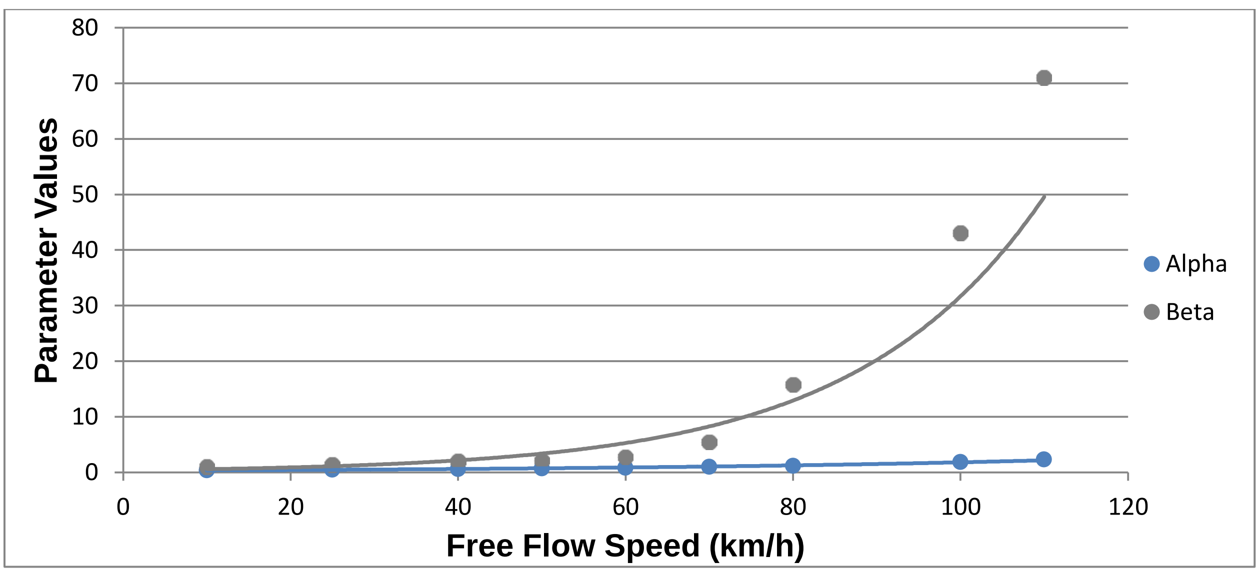

Figure 11: BPR Parameter Values

4.6.2. Conical Volume-Delay Function

The Conical VDF has just one independent parameter (α), on which the value of other parameter (β) is dependent. The α parameter of Conical function is somewhat similar to the one of BPR function. Spiess (1990, p. 157) clarified that the interpretation of the parameters used in both functions are identical. Besides, he added that if V/C ratio is less than 1, then the difference between estimated travel times using BPR and Conical VDF with the same value of parameter is apparently very small. Meanwhile, Spiess (1990, p. 155) also warned the value of α parameter is required to be any number larger than 1. However, according to Kalaee (2010, p. 70), smaller values for α makes the estimated travel time more sensitive to V/C ratio.

Same as BPR parameters, Horowitz (1991, p. 12) once again recommended different sets of Conical parameter (α) values for freeway and multilane highway. Using the similar approach, non-linear regression is applied on both sets of α value, such that the parameter values are consistent with all road facilities. The final values of α parameter are presented in *Table 4 and* a graphical representation in relation to the free flow speed is displayed in *Figure 12*.

Table 4: Conical Parameter Values

Facility Type	Free Flow Speed (km/h)	Alpha (α)	Beta (β)	Approach
Freeway	110	16.73	1.03	Non-linear regression of "freeway" set
Highway class-I	100	14.63	1.04	
Highway class-II	80	11.19	1.05	
Urban Street class-I	70	7.10	1.08	Original values of “multilane highway” set
Urban Street class-II	60	4.00	1.17	
Urban Street class-III	50	4.00	1.17	
Urban Street class-IV	40	2.73	1.29	Non-linear regression of "multilane highway" set
Minor Urban Street	25	1.77	1.65	
U-turn	10	1.15	4.25	

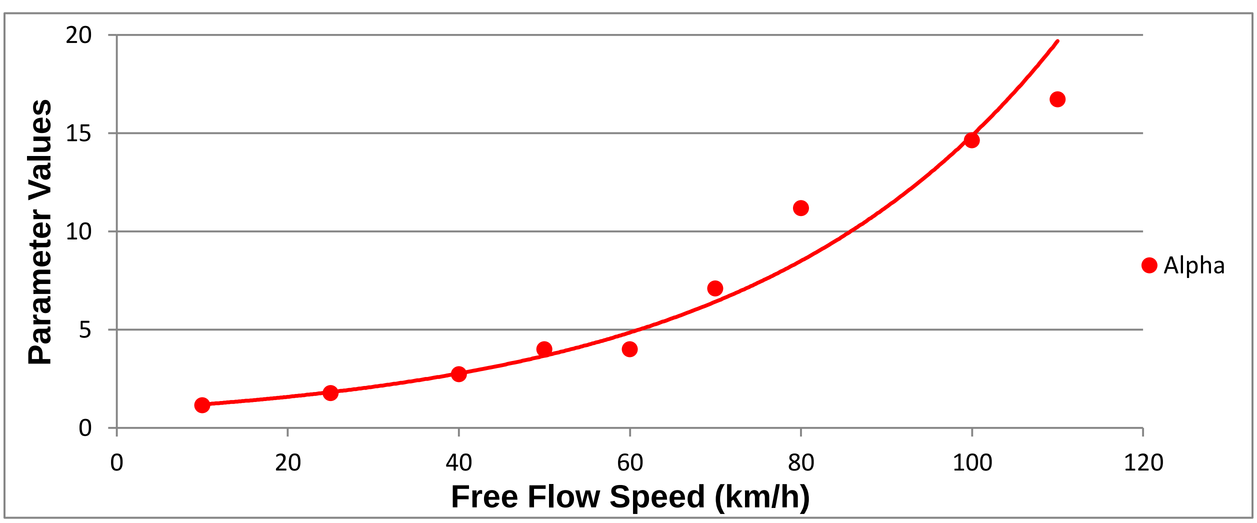

Figure 12: Conical Parameter Values

4.6.3. Akcelik Delay Function

The Akcelik volume-delay function contains one parameter, which is usually referred to as J. Akcelik (1991, p. 57) presented a table of different values for J, which varied from 0.1 to 1.6 with respect to different road facilities. However, Kalaee (2010, pp. 70–71) noted that the parameter values originally suggested by Akçelik are way higher than they actually should be. She also implied that smaller values of J parameter make estimated travel time less sensitive to the V/C ratio, in contrary to the parameters of BPR and Conical functions.

Transportation Research Board (2000, p. 30_8) presented a list of values for J parameter depending on specific road facility type and free-flow speed. In the framework of the network used in this study, it is possible to obtain parameter values for the first 7 road categories (from freeway to urban street class-IV) from that table. Therefore, in order to retain parameter values for remaining 2 road classes (minor urban street and U-turn), the non-linear regression method is followed. The final values of J are mentioned in *Table 5* and plotted in *Figure 13:*

Table 5: Akcelik Parameter Values

Facility Type	Free Flow Speed (km/h)	Parameter J (min^2/km^2)	Approach
Freeway	110	0.028	From the table presented in Transportation Research Board (2000, p. 30_8)
Highway class-I	100	0.007	
Highway class-II	80	0.004	
Urban Street class-I	70	0.079	
Urban Street class-II	60	0.180	
Urban Street class-III	50	2.876	
Urban Street class-IV	40	11.502	
Minor Urban Street	25	12.390	Non-linear regression
U-turn	10	53.087	

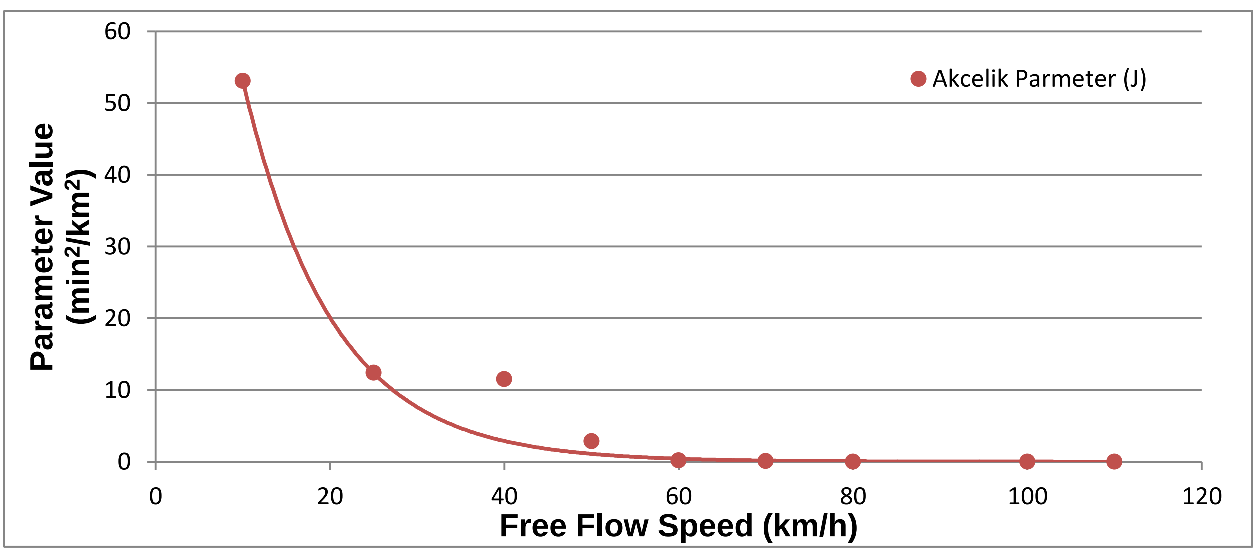

Figure 13: Akcelik Parameter Values

4.6.4. Logit-based Volume-Delay Function

There are 8 parameters in logit-based function, whose relationships are explained in *chapter 2.4.4*. However, the instances of practical application of logit-based VDF, in comparison with other delay functions, are very seldom. After spending decent effort into the research for parameter values of logit-based function, no credible references are found. At the end, it is settled for using the default values given in the TransCAD software package for logit-based parameters. These parameter values are presented in *Table 6*.

Table 6: Parameter Values of Logit-based VDF

Parameters	**Values**
a_1	0.9526
a_2	1
a_3	3
a_4	3
b_1	0.04046
b_2	500
b_3	3
b_4	3

4.7. Convergence and Relative Gap Criteria

User equilibrium is an iterative process and the quality of the solution relies upon the degree of the convergence. Therefore, the DUE model needs to achieve a satisfactory level of convergence around the equilibrium condition. In case, the assignment solution is far away from the equilibrium condition, the consequences of such ill-converged algorithms are manifold. For an example, the small and localized changes in network properties (such as free flow speeds or link capacities) will be reflected all over the network making the solutions unrealistic for impact assessment. In order to ensure that the algorithm will not stop before reaching a stable solution, a well-designed convergence criterion is chosen. The convergence criterion is comprised of two parts, one is the number of iterations and another is the measure of convergence.

While there are different possible measures of convergence exist, Rose et al. (1988, p. 271) reviewed a variety of them and recommended a particular one called relative gap. Relative Gap measures the gap between the current assignment solution and the optimal solution (ideal shortest-route time for all O-D pairs and all departure intervals). In other words, the relative gap is an estimate of the "distance" between the current solution and the equilibrium solution. U.S. Department of Transportation, Federal Highway Administration (2012, pp. 2_7 & 2_8) provided the following formula in order to calculate relative gap:

The typical definition of the total relative gap is as follows:

$$rel_{gap} = \frac{\sum_t \sum_{i \in I} \left(\sum_{k \in K_i} f_k^t \tau_k^t \right) - \sum_t \sum_{i \in I} d_i^t u_i^t}{\sum_t \sum_{i \in I} d_i^t u_i^t}$$

Where,

T = set of all departure time-intervals

t = departure time-interval, $t \in T$

I = set of all origin-destination trip pairs

i = origin-destination trip pair, $i \in I$

K_i= set of all used routes for origin-destination pair i

k = used route for origin-destination pair i, $k \in K_i$

f_k^t= flow from used route k at departure time-interval t

$\tau_k{}^t$ = experienced travel time on used route k at departure time-interval

$d_i{}^t$ = total flow from origin-destination pair i at departure time-interval t

$u_i{}^t$ = shortest route travel time from origin-destination pair i at departure time-interval t

When the relative gap is lower than a pre-defined tolerance level, the DTA solution is assumed to have converged sufficiently to an equilibrium solution. After a few test runs, it was found that a relative gap of 0.0001 leads to sufficient convergence in the inner iteration. Meanwhile, for outer iteration, a value of 0.01 was set for the relative change of Node-Time-Arrival matrix. The algorithm behind these two-level iterations was discussed in *chapter 3*. In line with the specific objective of this research, the assignment procedure had to be performed many times, so that sufficient assignment-outputs were obtained in order to conduct a comparative analysis. During all these assignment runs, the identical convergence criteria were chosen. These convergence criteria are presented in *Table 7*.

Table 7: Convergence Criteria

Iteration Level	**Maximum Number of Iterations**	**Convergence Criteria**	**Convergence Criteria Type**
Inner Iteration (FW Assignment)	100	0.0001	**Relative Gap**
Outer Iteration (Node-Time-Arrival)	100	0.01	**Relative Change of NTA matrix**

5. Result and Comparative Analysis

The dynamic traffic assignment is performed a total of 8 times, with 4 different volume-delay functions and each VDF for two peak-hour periods. Among them, 6 assignment runs (the ones with BPR, Conical and Akcelik function) fulfill the specified convergence criteria, i. e., converged successfully. However, the remaining two assignment runs (the ones with logit-based function) fail to reach to the convergence. Since these assignments remained unconverged, the assignment-outputs with logit-based VDF are not suitable for this comparative study. Hence, the logit-based function dependent assignment-outputs are omitted from further analysis.

The successfully-converged 6 assignment runs produce 4 resultant simulation outputs, which are traffic flow, assignment-speed, travel time and volume/capacity ratio respectively. These assignment-outputs are generated for each specific time-interval (10-minute time slice). As mentioned in *chapter 4.2* only final 6 time-intervals are considered appropriate for analysis, leaving the initial half-hour period for network-loading. Therefore, a total of 12 assignment-output sets from both morning and afternoon peak-hours are aggregated into three larger sets of assignment-output, each with BPR, Conical and Akcelik functions respectively. This data filtering and manipulation is performed with the help of the data analysis software package R. The data aggregation process is illustrated in *Figure 14*.

The comparative analysis is performed in terms of 9 indicators. Among them, one is qualitative indicator and the remaining 8 indicators are quantitative. These indicators and their indicated properties are presented in *Table 8*.

Table 8: Indicators and Indicated Properties

Type	Indicators	Indication of
Qualitative	Traffic Flow vs. V/C Ratio	Congestion
Quantitative	Level of Congestion	
	Speed vs. V/C Ratio	Speed
	Average Assignment-Speed	
	Volume vs. Count	"Goodness-of-Fit"
	Travel time	Travel time
	Convergence	Computational Efficiency
	Model Run-Time	
	VMT vs. VHT	System Performance

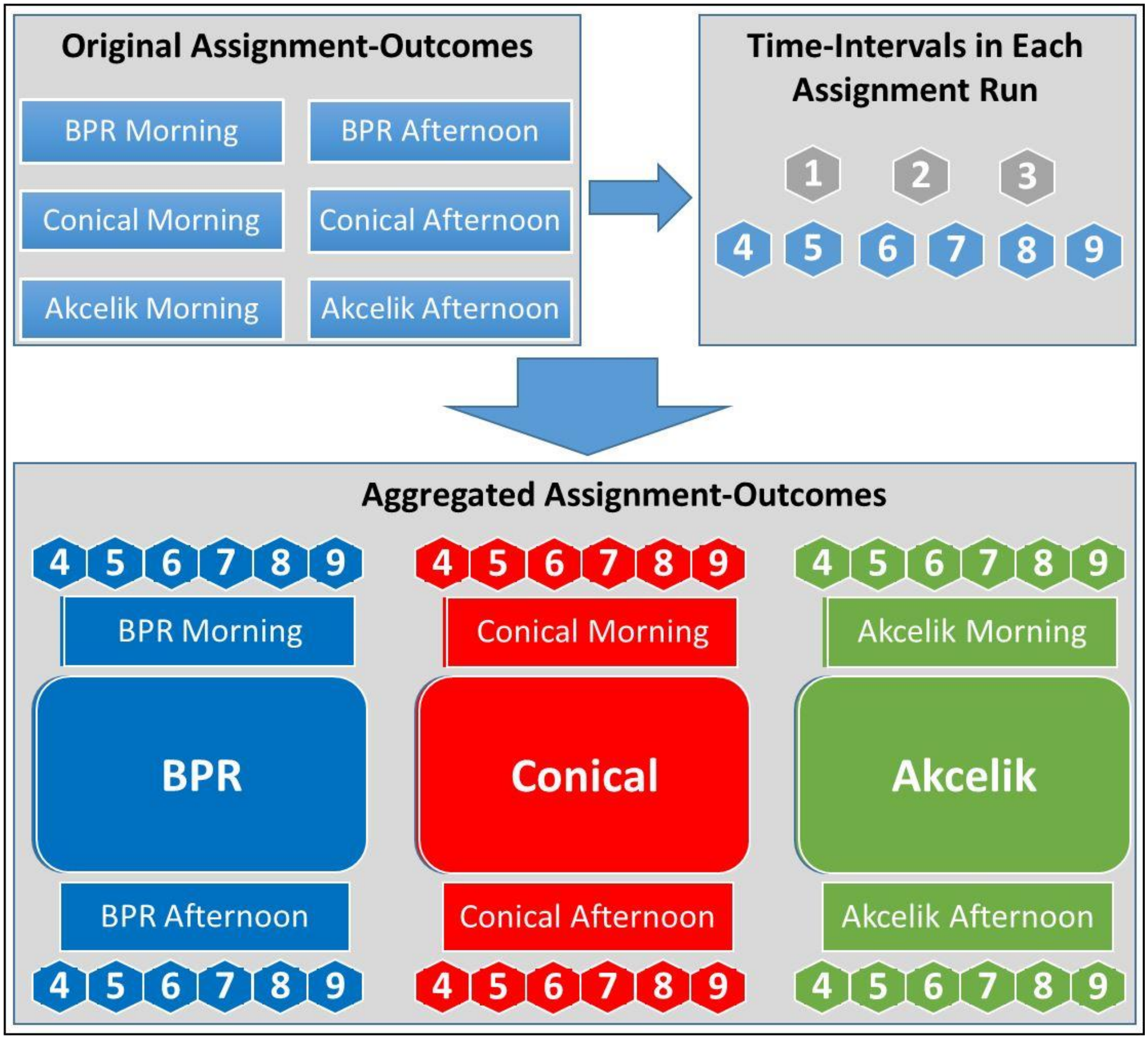

Figure 14: Aggregation of Assignment-Outputs

Each of the three large aggregated assignement-outputs (BPR, Conical and Akcelik) illustrated in Figure 14 are tested with the 9 indicators mentioned in Table 8. The results of each analysis are presented in the following sub-chapters, which are later synthesized as a final comparative analysis at the end of the chapter.

5.1. Visualization of Traffic Flow vs. V/C Ratio

An overview of the overall assignment at each individual time-interval can be investigated visually with a traffic flow vs. V/C ratio map. As an example, such maps of the 6th time-interval of the morning peak-hour (7:20 - 7:30) and afternoon peak-hour (17:20 - 17:30) are displayed in *Figure 15* and *Figure 16* respectively. The traffic flow vs. V/C ratio maps for the remaining time–intervals can be found in *Appendix A*.

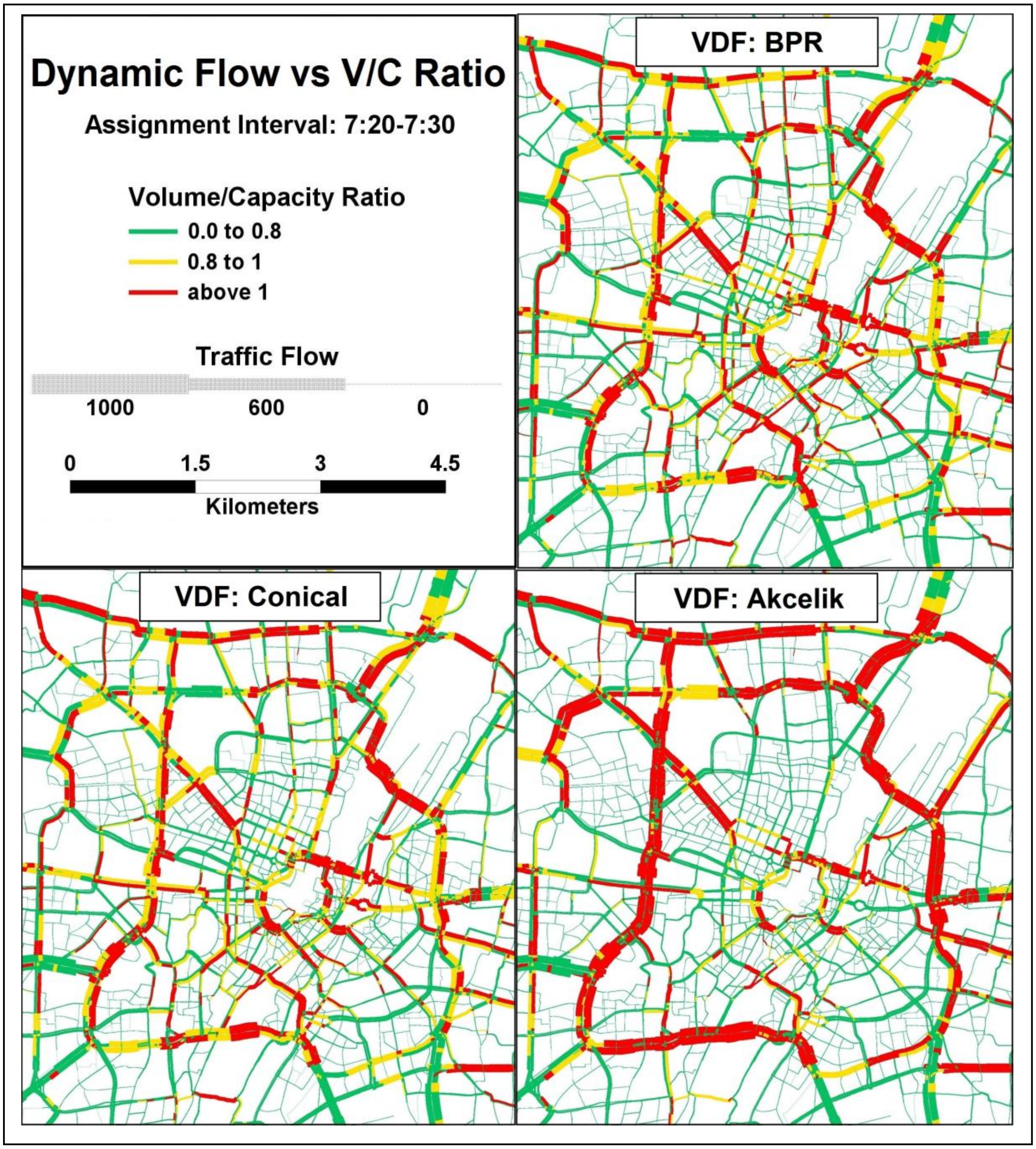

Figure 15: Traffic Flow vs. V/C Ratio for a Morning Peak-Hour Interval (7:20 - 7:30)

The side-by-side comparison of “traffic flow vs. V/C ratio” maps demonstrates that the assignment-outputs with BPR and Conical VDF are not quite different. However, the assignment with Akcelik function produces significantly more congestion in the network, particularly for two link categories (Urban Street class-I and Urban Street class-II), where the intersections are densely located and the volume are comparatively higher. The figures also give an impression that the morning peak-hour (7:20 - 7:30) time-interval is slightly more congested than the afternoon peak-hour (17:20 - 17:30).

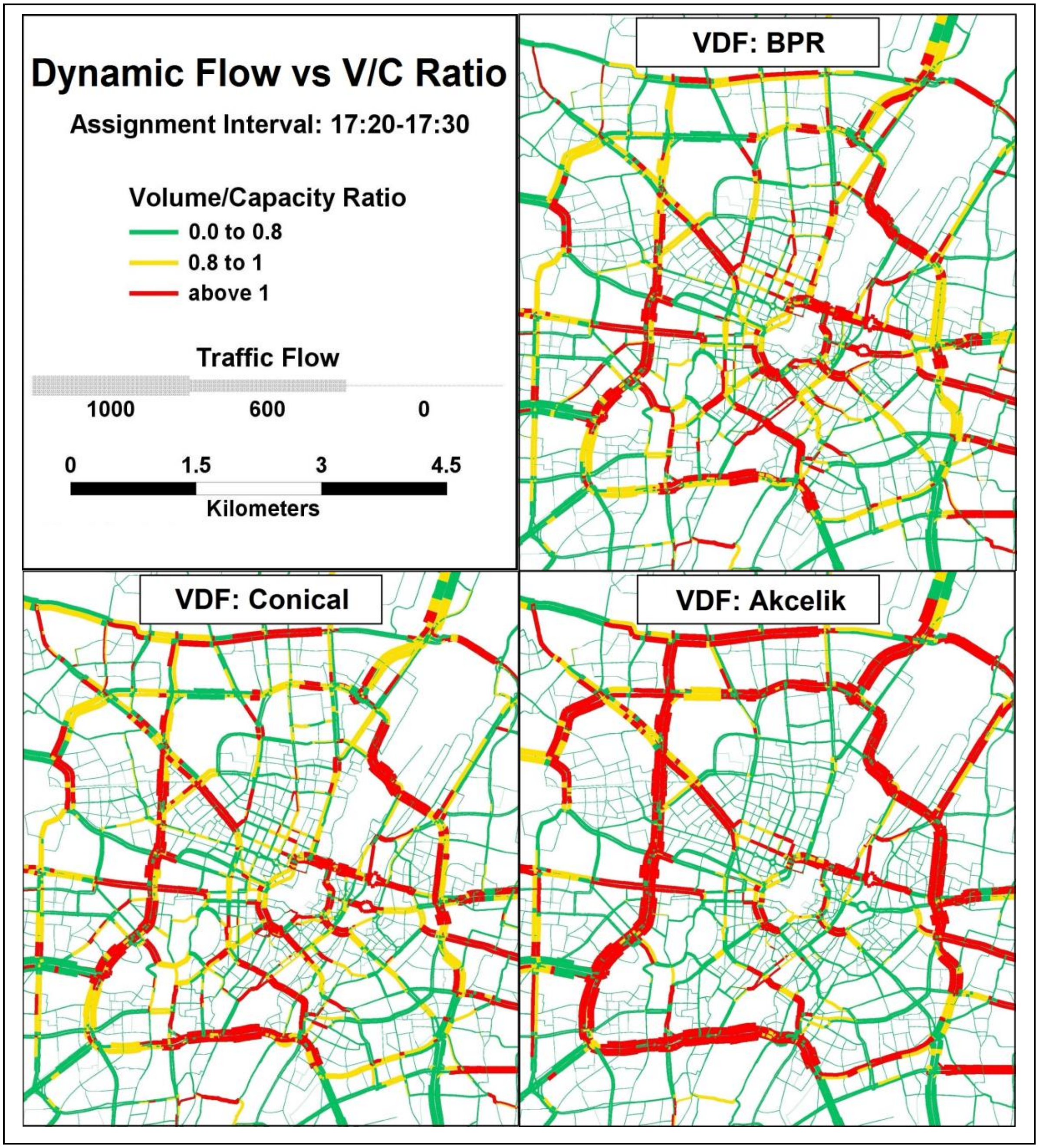

Figure 16: Traffic Flow vs. V/C Ratio for an Afternoon Peak-Hour Interval (17:20 - 17:30)

5.2. Speed vs. V/C Ratio

The comparative analysis of “speed vs. V/C ratio” explains how rapidly the speed drops with respect to the increasing V/C ratio. Mtoi and Moses (2014, p. 145) noted that a volume-delay function tends to perform differently in different facility types, especially in a congested network, For an instance, the change in congestion will have different impact on travel speed for a signalized arterial link compared to a freeway link. Therefore, the “speed vs. V/C ratio” analysis performed in this research is specific to each road facility type, as illustrated in *Figure 17* and *Figure 18* .

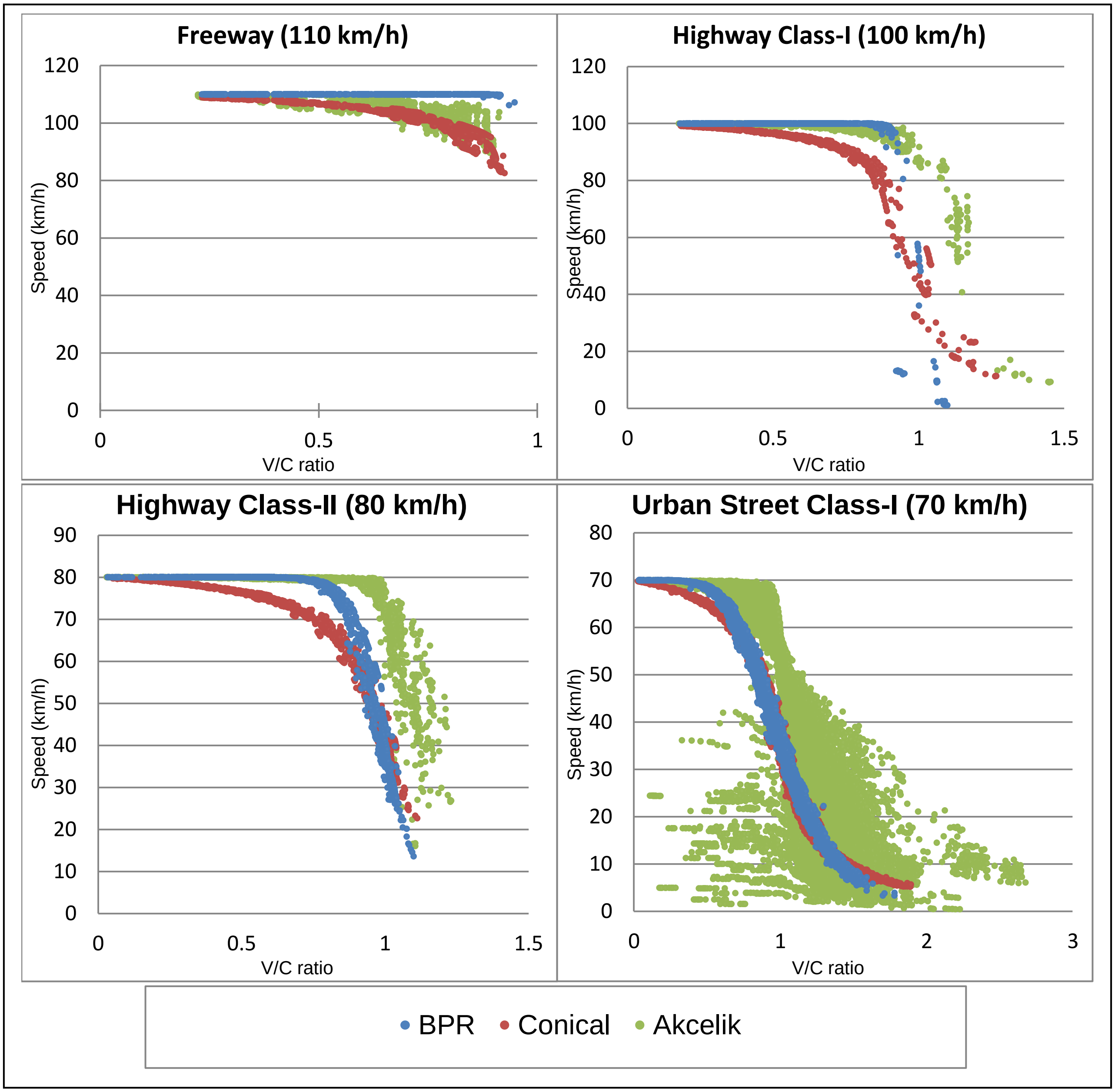

Figure 17: Speed vs. V/C Ratio for Road Categories 1-4

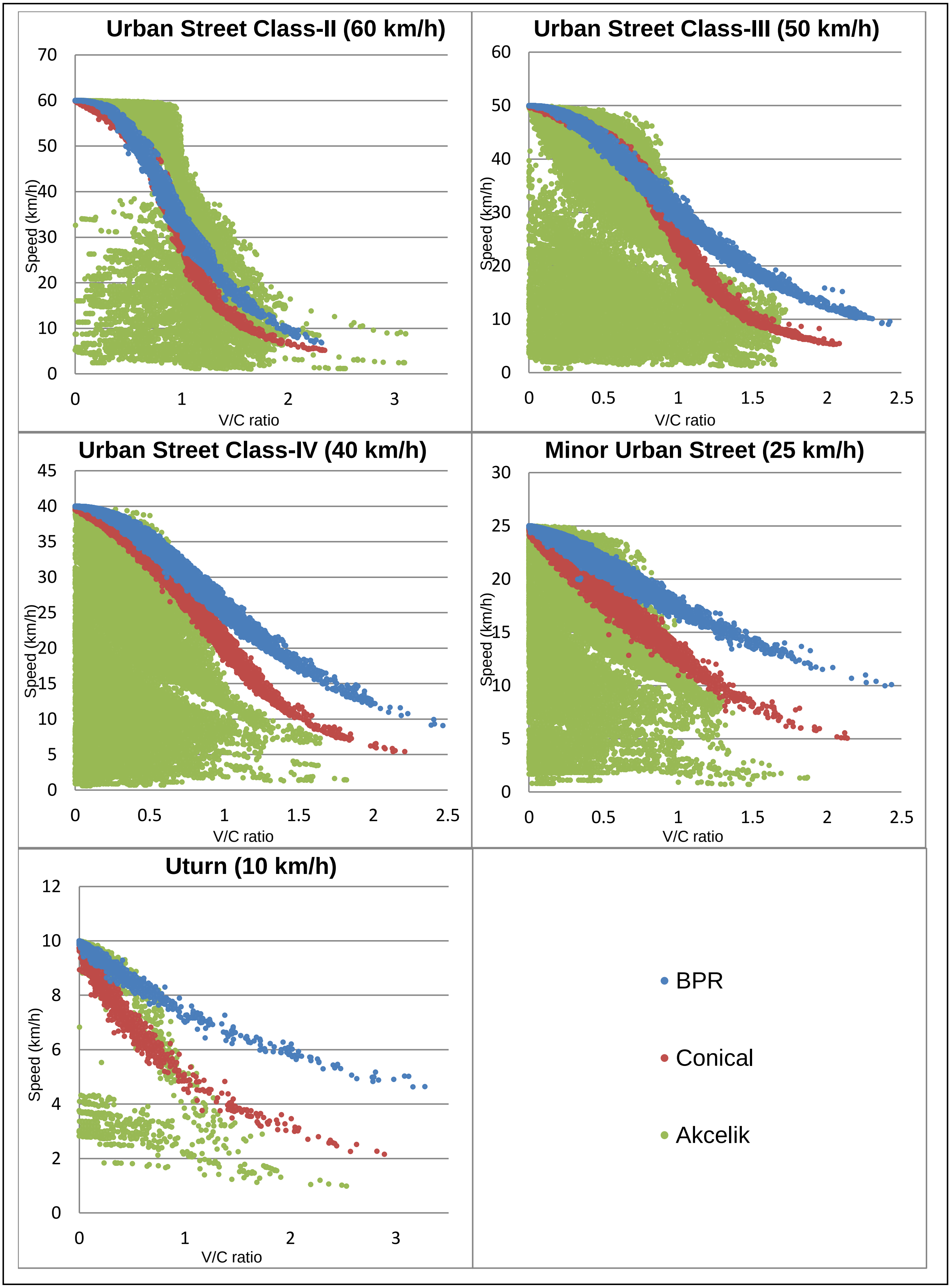

Figure 18: Speed vs. V/C Ratio for Road Categories 5-9

Figure 17 and *Figure 18* exhibit that the speed--curve of Conical function is steeper in comparison with the BPR function, which tends to show a bit flat curve.

On the other hand, the speed curve of Akcelik function exhibits two different behaviors depending on the road facilities, which are explained below:

- In higher road categories, the Akcelik speed curve is a bit flat likewise the BPR speed curve. This phenomenon is also consistent with the Figure 19 of Average Assignment Speed analysis where the average speed of Akcelik shows a sudden spike after FFS> 80 that it catches up the BPR.
- In lower road categories, the Akcelik speed curve appears to be split in two halves. The upper part is responsible for the regular link delay, same as the BPR and Conical curve. However, the lower part creates an unique traffic condition like "low speed with low V/C ratio". This part of the curve is believed to be caused by the intersection delay, where the vehicles wait at the signalized intersection even with an apparently empty link.

5.3. Average Assignment-Speed

The average assignment-speed analysis describes the extent of the reduction of average assignment-speed from the original free-flow speed (FFS). This change of speed can be observed by each road category as the average assignment-speed is plotted against the FFS.

The *Figure 19* displays that the average assignment-speed curve with BPR and Conical behaved quite similar below the FFS of 80 km/h. However, the speed with Conical VDF falls, when the FFS reaches to 80 km/h and higher. This incidence can be explained by the first 3 charts of *Figure 17*, which show that the speed curve with Conical is significantly below the BPR curve. On the other hand, the average assignment-speed of Akcelik function is lower than the ones of both BPR and Conical within the FFS range of 25 km/h and 70 km/h, nevertheless, it catches up the speed of BPR afterwards. That dip in the speed curve of Akcelik function is largely credited to the intersection delay. The assignment-speed of all three volume-delay functions hits the lowest at the FFS of 70 km/h (Urban Street class-I).

Figure 19 also shows the standard deviation in the distribution of assignment-speed values. The highest standard deviations for all three functions are also found again at 70 km/h FFS mark, which revealed the presence of extremely low speed values in Urban Street class-I, therefore the overall average speed dropped for in this domain.

The exact values for average assignment-speed and associated standard deviation are provided in *Appendix B*.

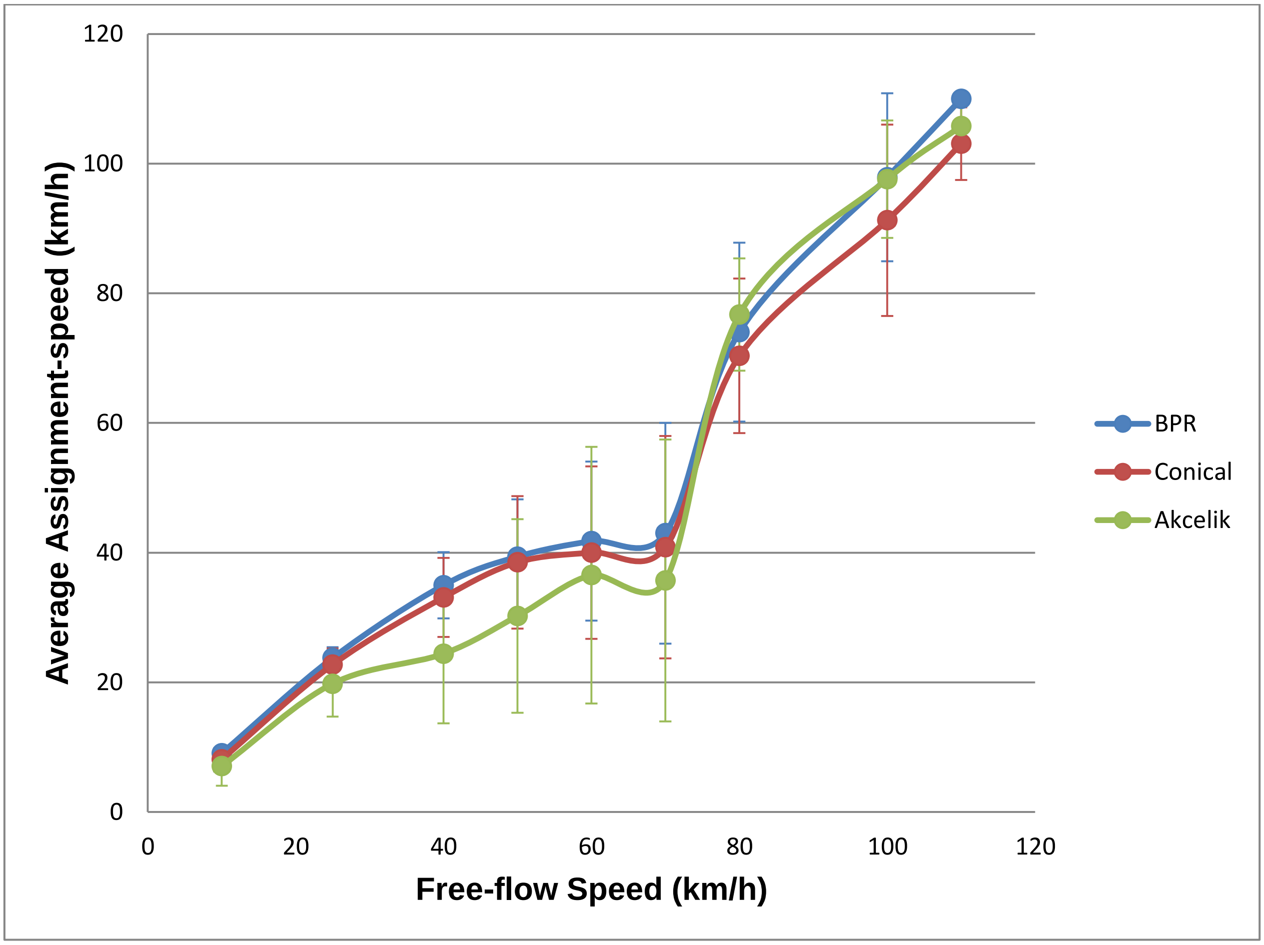

Figure 19: Distribution of Average Assignment-speed

5.4. Level of Congestion

The level of congestion analysis demonstrates the magnitude of congested networks in three assignment-outputs. This analysis is a quantitative representation of the visual examination of *chapter 5.1*. In this analysis, the V/C ratios of all the links are classified with a range of 0.25, and then the total length of all the links within each category are measured and the respective percentages were calculated.

The *Figure 20* illustrates that the assignments with BPR and Conical functions have almost the same level of congestion across the spectrum, as it is also visually evident from *Figure 15* and *Figure 16*. However, the assignment-result with Akcelik function produces substantially high volume of both low-level-congestion (V/C ratio < 0.5) and extreme-level-congestion (V/C ratio > 1.25). In contrary, the amount of mid-level-congestion (0.5 < V/C ratio < 1.25) of Akcelik assignment is very low compared to the ones of BPR and Conical.

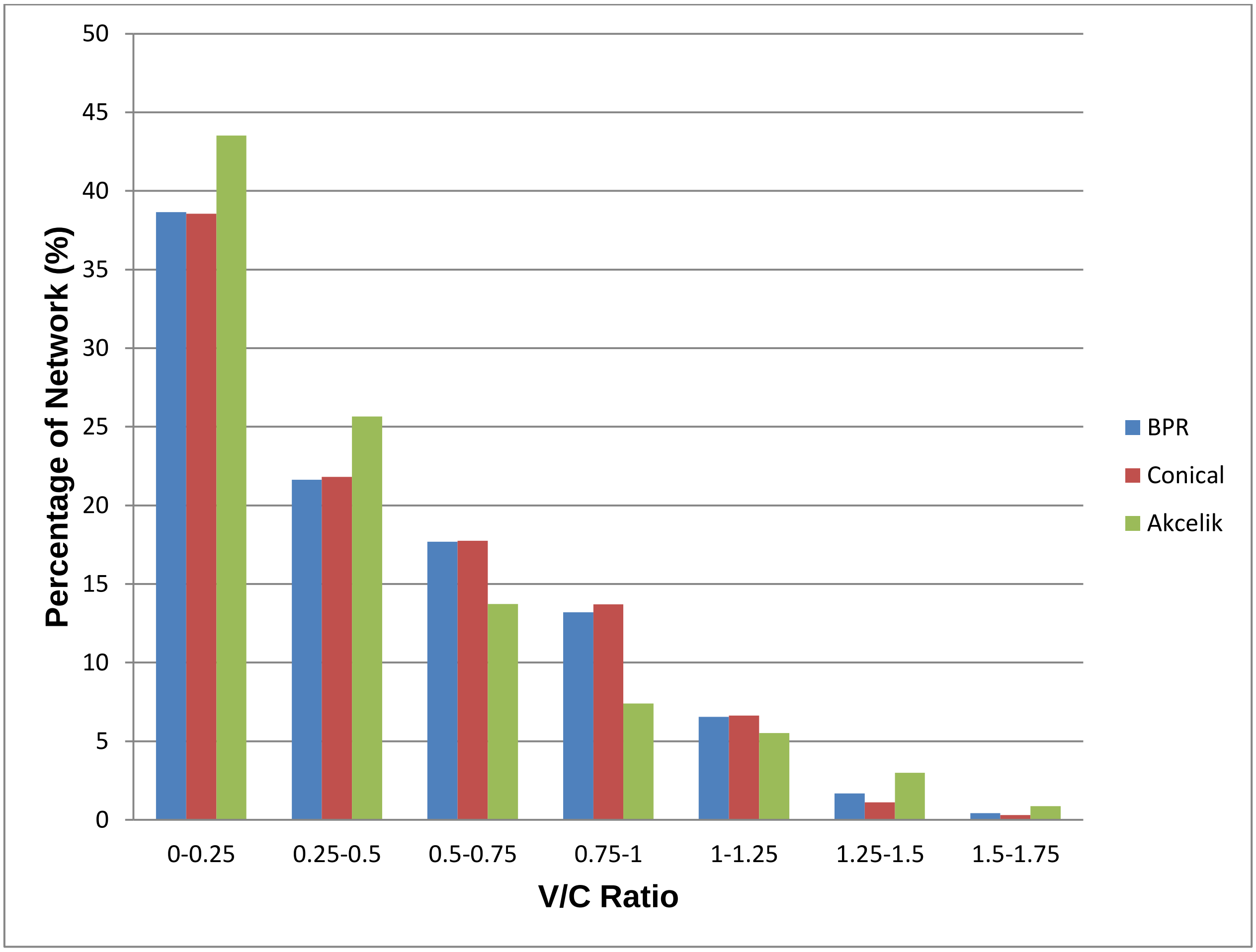

Figure 20: Level of Congestion

The total distance for each V/C ratio range and associated percentages are provided in *Appendix C*.

5.5. Volume vs. Count

The volume vs. count analysis checks the accuracy of simulated volume with respect to the empirical traffic count. This analysis is conducted based on the BASt traffic counts illustrated in *Figure 5*. The assignment volumes are plotted against the actual traffic counts in *Figure 21*, so that the exact traffic count values fit within a diagonal line. *Figure 21* shows the assignment volumes of all three assignment-outputs are randomly scattered around the diagonal line.

The figure indicates that it is difficult to draw a comparative evaluation, as the volume values of all three VDFs are randomly distributed, but in an equal manner. However, a notable point is that there is no case of any extreme value, i.e., all values remain within a distant range.

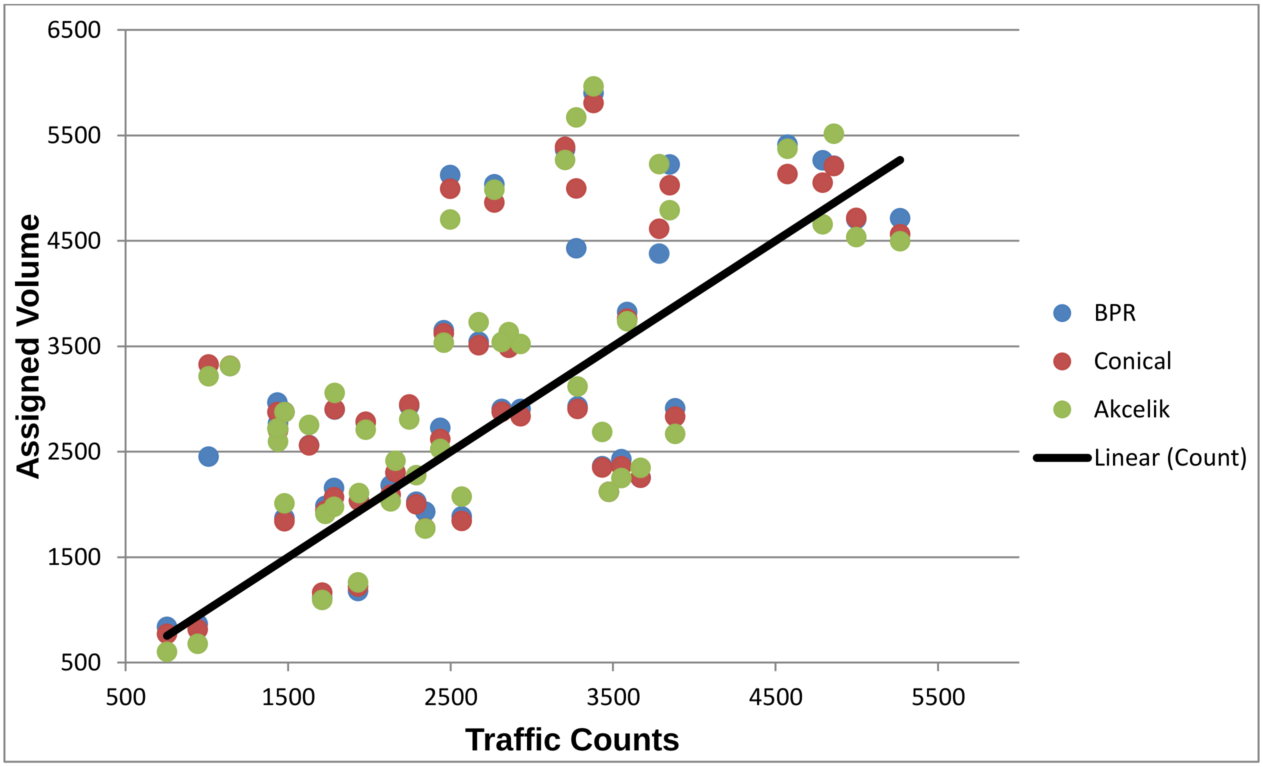

Figure 21: Assigned Volume vs. Traffic Count

All traffic counts and assignment flow values used in this analysis can be found in *Appendix D*. The irregular distribution of traffic volume values of all three VDFs from *Figure 21*, is also evident from their respective RMSE values, presented in *Table 9*. The RMSE values are specific to each road facility for which traffic count data were available.

Table 9: RMSE Values of Links with Traffic Counts

Road Facility	Free-Flow Speed	Number of Counts	%RMSE		
			BPR	Conical	Akcelik
Freeway	110	20	50.8	51.6	50.1
Highway class-I	100	16	32.2	33.5	39.8
Highway class-II	80	8	23.3	23.4	25.8
Urban Street class-II	60	4	36.3	33.9	36.0

Table 9 displays that the RMSE values of all three VDFs are a bit higher than the acceptable bar, particularly for the freeways. However, this result does not undermine the reliability of this research, because the objective of this research was to compare the different performances of different VDF, not to develop a perfectly realistic model. In any case, the model flow however is perfectly able to follow the real traffic counts, even may not with such accuracy.

5.6. Travel time

The travel time analysis compares model travel time of three assignment-outputs with the empirical travel time data. In order to conduct this analysis, 30 points with their specific coordinates were selected from the network. These origin and destination points form 15 shortest-path routes among them. The travel time of these 15 routes were calculated separately for morning and afternoon peak-hour for each assignment-output. Therefore, each of the 3 assignment-outputs has altogether 30 travel time data. On the other hand, the empirical travel time data were collected from the Google Map, which provided a travel time range with a lower and an upper bound for a specific departure time. These Google Map data were collected by setting the departure time as the 4th time-interval of each peak-hour (7:00 for morning peak-hour and or 17:00 for afternoon peak-hour), which is the beginning of the analysis period.

Figure 22 displays that the BPR and Conical function estimate almost identical travel time. However, the travel time of Akcelik function is constantly higher than the other two, which is likely a result of intersection delay. Besides, it is quite evident that the travel time estimated by the Akcelik function fits into the range of Google Map travel time the best. In contrast, both BPR and Conical function tend to underestimate the travel time continuously, with respect to the Google Map.

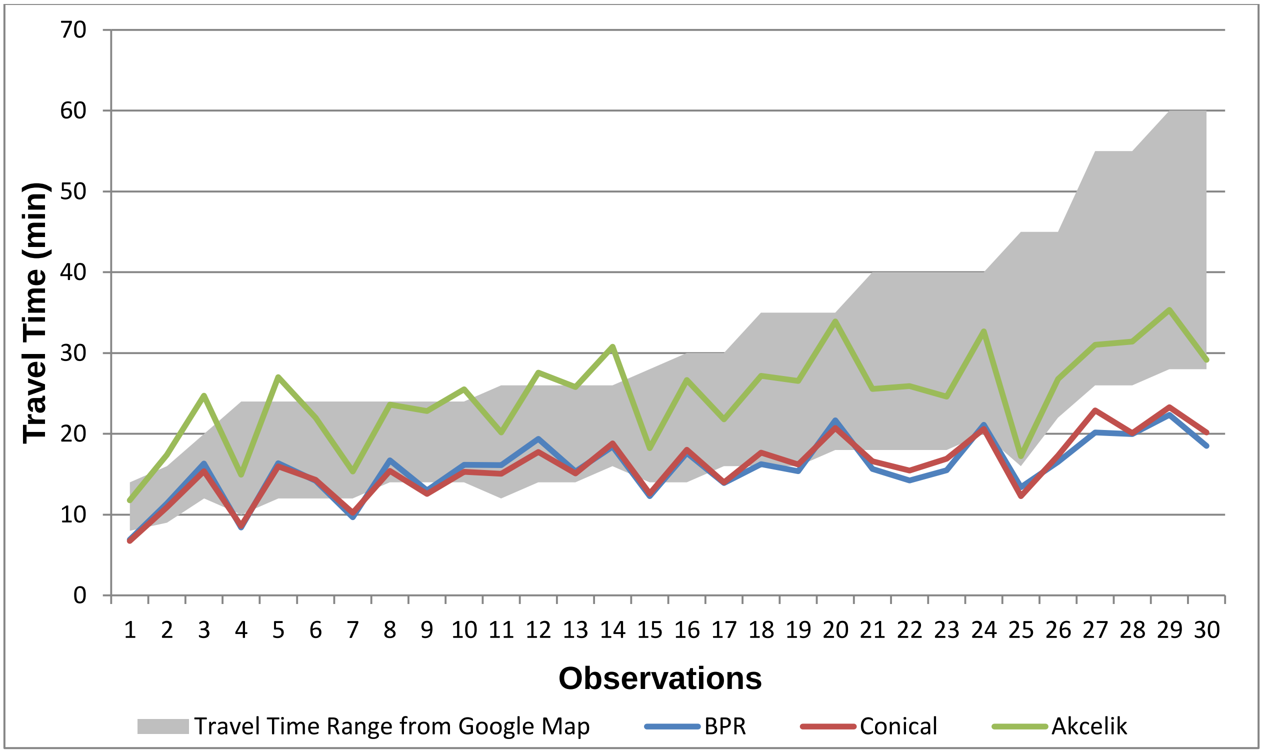

Figure 22: Travel time Analysis

For the exact values of all travel time used in *Figure 22*, Appendix E needs to be looked at.

5.7. Convergence

The convergence comparison investigates how many iterations each assignment run requires, i.e., how rapidly an assignment solution reaches to convergence for each volume-delay functions. The particular DTA algorithm used in this research has two levels of iterations, as explained in *chapter 3*. The number of outer iterations (node-time-arrival) and inner Iterations (FW assignment) are plotted across X and Y axis respectively.

Figure 23 illustrates that the assignment run with BPR function requires the least number of iterations of both outer and inner level. On the other hand, both Conical and Akcelik functions need an extra outer iteration compared to BPR function. Moreover, during the first 3 outer iterations, both Conical and Akcelik functions failed to converge the inner-level (FW) assignment solution within the predefined limit, since the predefined maximum number iterations was limited to 100, as described in *Table 7*.

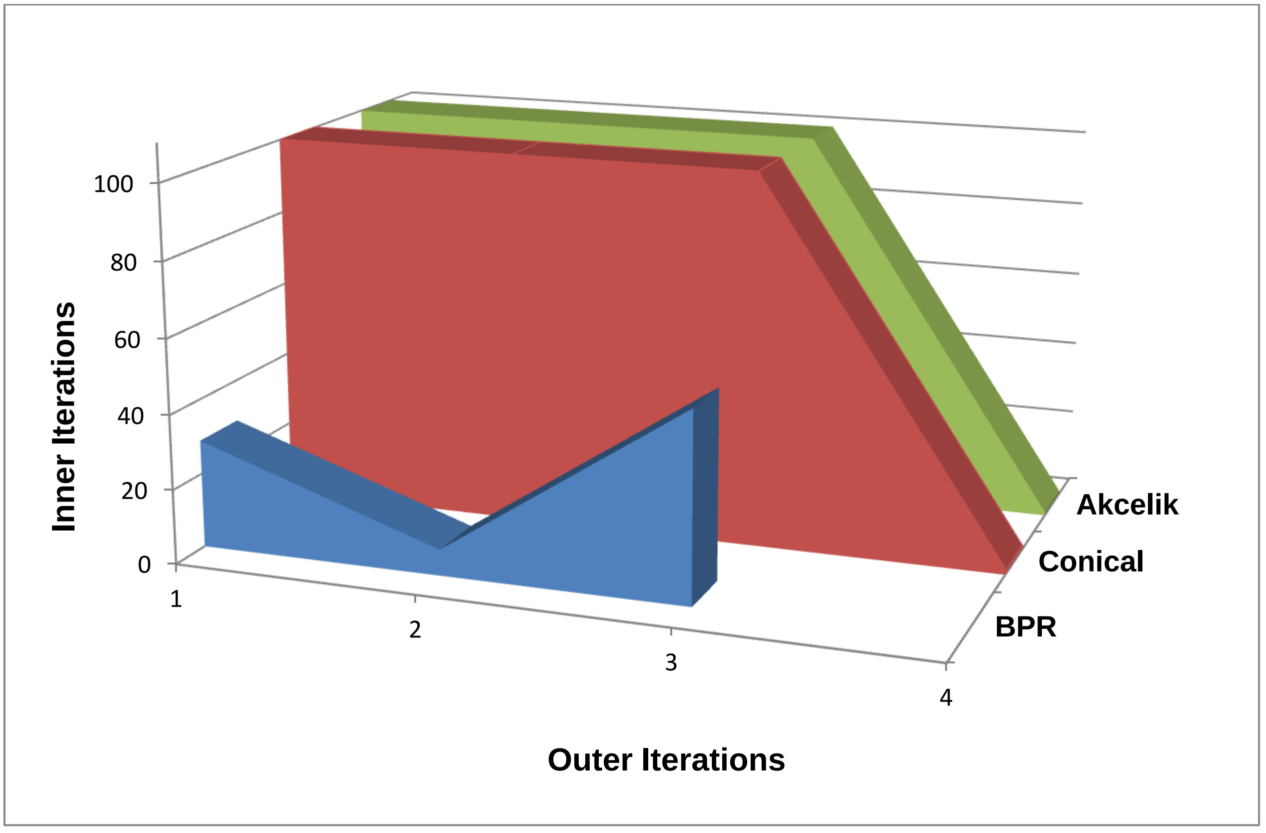

Figure 23: Number of Iterations Required for Convergence

5.8.Model Run-time

The run-time inspection compares time required for each assignment run to achieve convergence. *Figure 24* shows that the assignment with BPR function took almost one-third of the time required by other two assignment runs. Among them, the one with Conical function required the longest run-time.

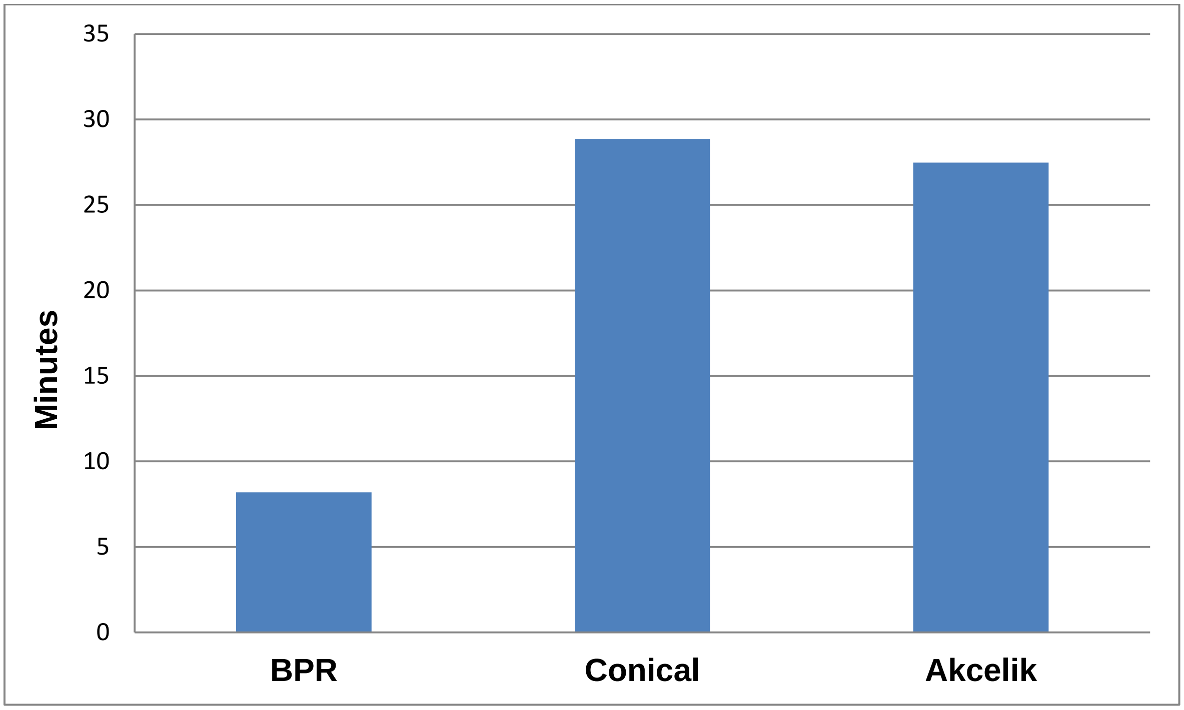

Figure 24: Model Run-time

5.9. VMT vs. VHT

The vehicle-miles travelled (VMT) represents the total distance travelled by all vehicles during the full assignment period which is recorded in miles. On the other hand, the total travel time spent in hours by all vehicles during the assignment period is known as the Vehicle-Hours Travelled (VHT). Both VMT and VHT are measurements of system-wide cost.

Figure 25 illustrates that despite having recorded the least miles in VMT, the assignment run with Akcelik function registers the most hours in VHT. This phenomenon is a result of the intersection delay component of Akcelik function, which escalated the travel time of the whole system. On the other hand, the assignment runs with BPR and Conical VDF produce balanced outcome in both VMT and VHT.

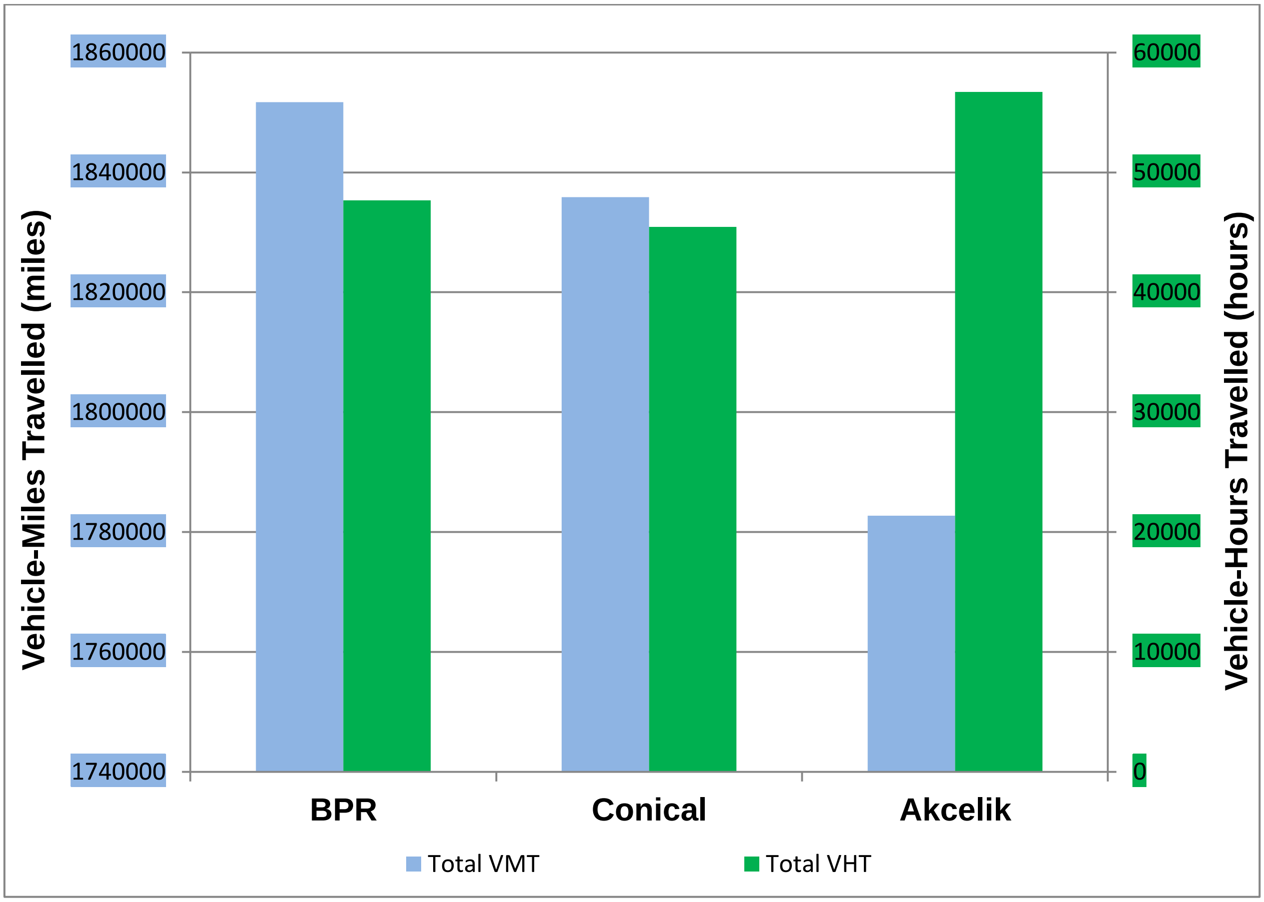

Figure 25: Total VMT & Total VHT

5.10. Comparative Overview

The comparative overview of the above analysis is presented in *Table 10*, which exhibits that different volume-delay functions perform variably across different indicators.

Three congestion related indicators (Visualization of flow vs. V/C ratio, Level of Congestion and Travel time) indicate that BPR and Conical functions produce almost similar outcome, but Akcelik function generates more congestion in the network.

The Speed is represented by two indicators (Speed vs. V/C Ratio and Average Assignment-speed) which demonstrate that the speed of BPR function and Akcelik function (for higher road facilities) drops gently with respect to the increasing V/C ratio, thus they produce higher average speed. On the contrary, the speed of Conical VDF is much more sensitive to the V/C ratio. In particularly, for higher road facilities, the conical speed curve drops down too early, thereby the average speed of conical function at the higher road categories is lower. On the other hand, for the lower levels of road facilities along with intersection delay, the speed curve of Akcelik function drops irrespective of the V/C ratio, which leads to a dip in the average speed for those road categories.

Regarding the "Goodness-of-Fit" of the model, the “Volume vs. Count” indicator produced equally random result that failed to replicate a reasonable fit with the reality. Therefore, it has to be concluded that all three VDFs failed to simulate a close resemblance to the realistic traffic flow volume.

In relation to travel time indicator, the travel time of BPR and Conical functions are almost identical and both underestimate the travel time when compared with the actual Google Map travel time. On the other hand, the travel time of Akcelik VDF roughly manages to fit into the travel time range of Google Map. The higher estimation of travel time by Akcelik VDF is largely due to the intersection delay.

Table 10: Comparative Overview of Assignment-Outputs with Three Volume-delay Functions

Indicators	**BPR**	**Conical**	**Akcelik**
Flow vs. V/C Ratio	same as Conical	same as BPR	more congested than BPR and Conical
Level of Congestion	same as Conical	same as BPR	higher than BPR and Conical for V/C ratio < 0.5 and V/C ratio > 1.25, but lower for 0.5 < V/C ratio < 1.25
Speed vs. V/C Ratio	speed curve is a little bit flat	speed curve is steeper	•for higher road levels, speed curve is flat as BPR •for lower road levels, speed curve is split in two half
Average Assignment-speed	•higher than Conical (for FFS≥80) •lowest value at FFS = 70km/h. •highest standard deviation is at FFS = 70 km/h	•lower than BPR (for FFS≥80) •lowest value at FFS = 70km/h. •highest standard deviation is at FFS = 70 km/h	•lower than BPR and Conical (for 25km/h ≤ FFS ≤ 70km/h, catches up BPR (for FFS≥80) •lowest value at FFS = 70km/h. •highest standard deviation is at FFS = 70 km/h
Volume vs. Count	high RMSE values	high RMSE values	high RMSE values
Travel Time	•lower than Google Map travel time. •same as Conical	•lower than Google Map travel time. • same as BPR	•fits into the Google Map travel time range. •higher than BPR and Conical
Convergence	lowest number of iterations	same as Akcelik	same as Conical
Model Run-time	almost one-third of Conical and Akcelik	same as Akcelik	same as Conical
VMT vs. VHT	balanced result in both VMT and VHT	balanced result in both VMT and VHT	•lowest VMT •highest VHT

On the issue of computational efficiency, two indicators (Convergence and Model Run-time) confirm that Conical and Akcelik functions are computationally more challenging than BPR function.

Finally, the system performance indicator, VMT vs. VHT, demonstrates that BPR and Conical functions result fairly proportionate outcome, but the Akcelik function yields disproportionately high value of total system-time in contrast with very low total system mileage. This phenomenon once again endorses the result of the previously mentioned three congestion related indicators, i.e., the Akcelik function causes severe congestion at the intersections.

6. Conclusion

6.1. Limitations and Further Research Scopes

Limitations:

- The afternoon peak-hour assignment-output using BPR function showed unusually high travel time and barely any traffic flow in a particular corridor of the network. That corridor was comprised of 22 links, which were considered negligible in a network of 15542 links. As a result, those 22 links were excluded from the data before the analysis. Since the overall assignment solution was converged and reached to the equilibrium condition, this phenomenon is likely caused by a discrete and isolated error.
- In order to ease the computational burden, many local and residential level streets were excluded from the network, as described in chapter 4.4. This might have caused a minor effect in the assignment.
- The intersection data needed for the assignment-input were estimated based on the procedure explained in chapter 4.5, instead of using real intersection data.
- For the Adjustment factor (D_F) of Akcelik volume-delay function, a general value of 1.0 is assumed, which is only suitable for uncoordinated fixed-time signals. However, the Munich city also has many traffic-actuated signals and coordinated signals with favorable/unfavorable progression. Therefore, assuming a general value might have produced a biased assignment result.
- The original traffic demand data provided by the *Professorship*, contained car vehicles only. Therefore, the absence of other modes (such as heavy vehicle, bus, tram and bicycle) is a major limitation of this assignment model.
- The assignments with Logit-based VDF were failed to reach to convergence. The most likely theory is that the network and OD matrix were too large for the Logit-based VDF that it posed such an excessive computational burden. Therefore, the assignment problem with logit-based VDF could not able to converge within a feasible iteration limit.
- The model failed to simulate a close resemblance to the real traffic counts, as revealed by the volume vs. count analysis (described in chapter 5.5).
- Apart from the traffic counts on 12 locations and the travel time collected from Google Map, no other empirical data were available for the indicators.
- The original traffic demand data of the *Professorship* did not contain the external trips from/to outside the study area of the greater Munich Metropolitan model. It means that a large amount of long distance traffic went unaccounted for.
- Although the original traffic demand data of the *Professorship* was developed by following a strict scientific methodology, this demand may not reflect the real traffic condition with precise accuracy.
- The network capacity values were collected from the Highway Capacity Manual 2000. The HCM 2000 recommended values may not replicate the actual road capacity in the study area.

Further Research Scopes:

- The assignment with Logit-based VDF failed to convergence. It would be interesting to investigate the convergence issue of the Logit-based VDF.
- There is still room for further researches in comparing other unconventional VDFs.
- There is still scope for exploring the parameter sensitivity of the VDFs and studying the calibration opportunities.

6.2. Concluding Remarks

This thesis aimed to conduct a comparative analysis on the performances of different VDFs in a DTA model. Four VDFs were selected initially for this experiment, which are BPR, Conical, Akcelik and Logit-based VDF. As for the DTA model, the mathematical programming based DTA method available in TransCAD software package was chosen, because this particular DTA approach uses VDFs and requires less input-data than typical DTA models.

The DTA model was developed for the Munich city area by using the network from the *HERE Map data 2014*. On the other hand, the TAZ and associated traffic demand data were provided by the *Professorship of Modeling Spatial Mobility*. Both HERE network and the TAZ system of the *Professorship*, were modified adequately before put into application. The daily distribution of traffic demand in the study area was calculated by analyzing the traffic count data obtained from *Bundesanstalt für Straßenwesen (BASt)*. Based on this analysis, two peak hours were selected, one is at the morning and another is at the afternoon. Two OD matrices were created for two peak hours using the traffic demand data of the *Professorship*. Most of the network properties (such as capacity, FFS, intersection data etc.) and VDF parameter values were collected from the Transportation Research Board (2000), which is known as the *Highway Capacity Manual (HCM 2000)*. Finally, the Assignment procedure with each VDF was run twice (for two peak hours).

Among the four VDFs tested, only the assignments with logit-based VDF were found unconverged after a long iteration period. The remaining assignments with three VDFs (BPR, Conical and Akcelik) successfully converged to equilibrium condition. These six assignment-outputs associated with those 3 VDFs were aggregated and processed for the comparative study using the statistical analysis software package R.

In order to conduct the comparative analysis, 9 indicators (1 qualitative and 8 quantitative) were selected. These indicators indicate the performance of each VDF with respect to 6 model properties, which are congestion, speed, "goodness-of-fit" of the model, travel time, computational efficiency and system performance. Due to the lack of available empirical data, conducting a definitive evaluation was not possible for many of the indicators. In these cases, a general quantitative or qualitative comparison of the performances of three VDFs was presented. However, for some indicators such as travel time, convergence and run-time, deriving a comparative evaluation was possible.

The analysis results from different indicators exhibit a diverse picture on the performances of different VDFs. In terms of congestion, BPR and Conical functions develop almost analogous congestion scenario in the network, however Akcelik function produces heavy congestion, particularly around the intersections. Regarding speed, Conical VDF is more sensitive to the increasing V/C ratio, than BPR and Akcelik. However, the Akcelik function's speed slumps irrespective of the V/C ratio for the lower grade roads with signalized intersection. On the issue of travel time, BPR and Conical VDF both almost identically underestimate the actual travel time. In contrast, Akcelik VDF roughly manages to resemble the actual travel time, due to the addition of intersection delay. In relation to computational efficiency, BPR function is computationally more efficient than Conical and Akcelik VDFs. Regarding system performance, BPR and Conical functions result fairly balanced system performance, but the Akcelik function yields disproportionately high value of total system-time in contrast with very low total system mileage. Nevertheless, all three VDFs failed to simulate a close resemblance to realistic traffic flow volume.

From the result, it is quite obvious that different VDFs perform quite differently with respect to different indicators. Therefore, it is concluded that there is no VDF that can be regarded as "the best" universally. Rather, it depends on which indicators are priorities for a specific DTA model. Thereby, the VDF that performs the best in the specific indicators, relatively important to a model, should be considered as the most suitable VDF for that particular model.

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Appendix

Master's Thesis:

Comparative Study of Different Volume-Delay Functions in A Mathematical Programming based Dynamic User Equilibrium Traffic Assignment

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Submission Date:	16.09.2018

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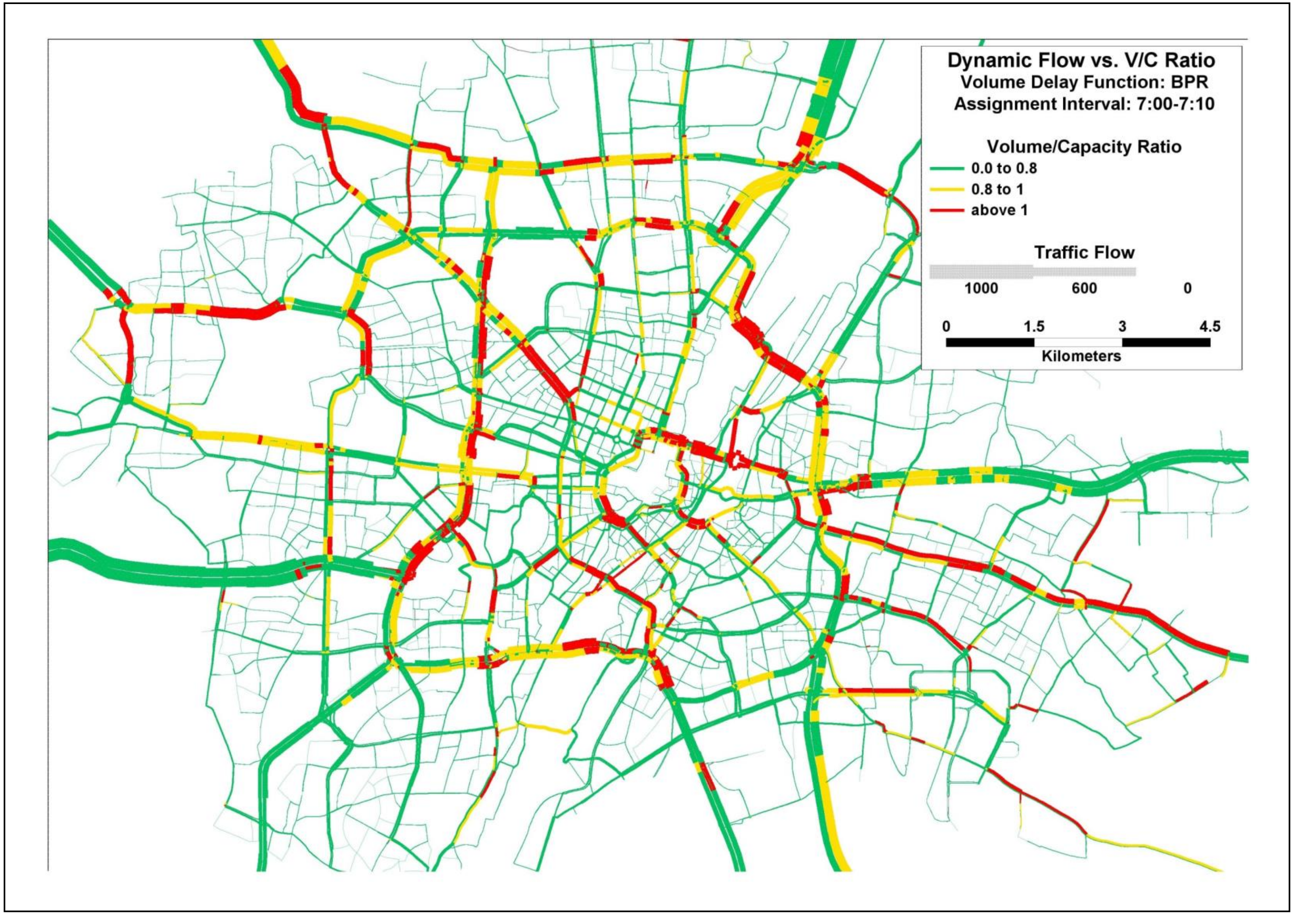
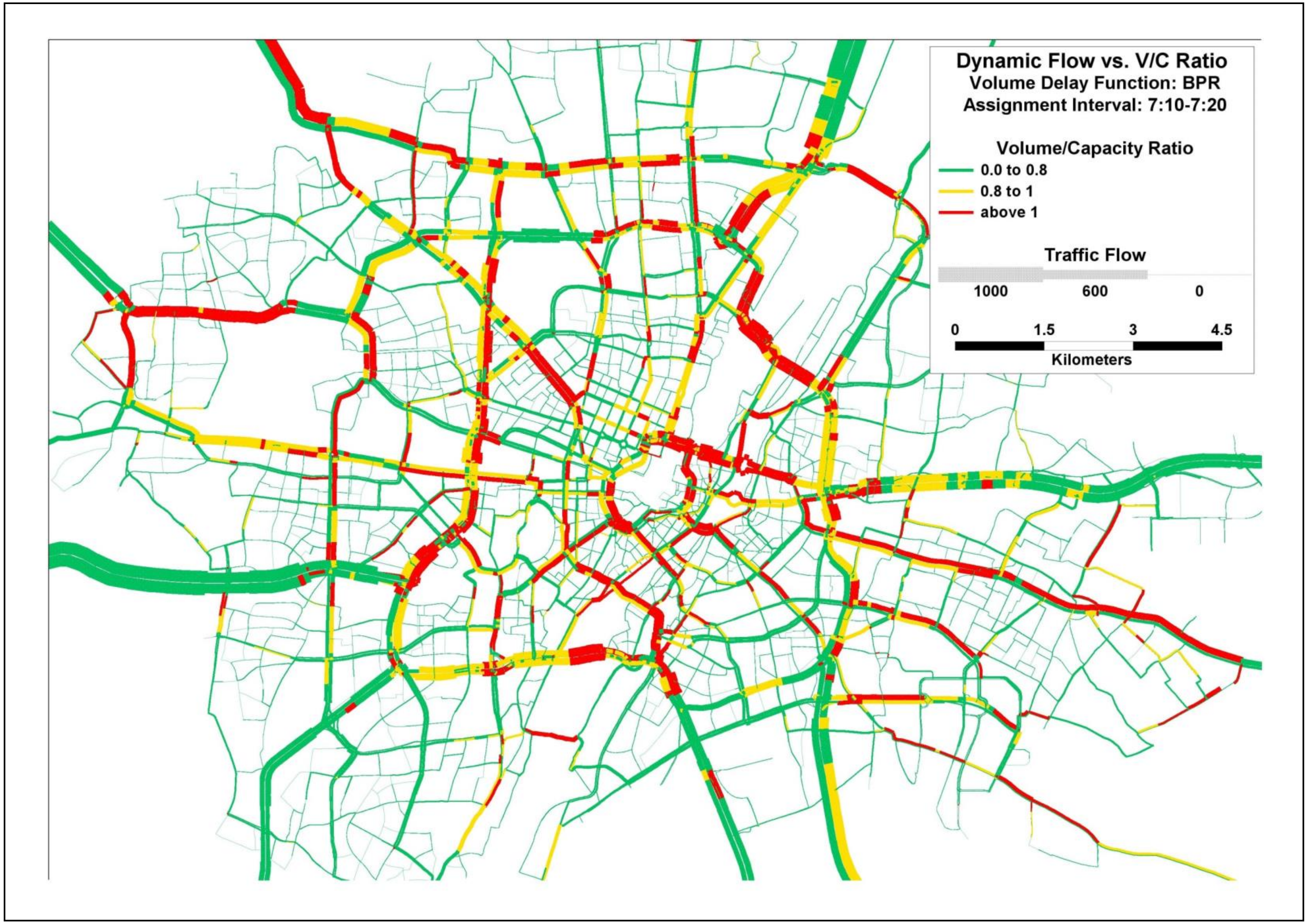
A. Dynamic Traffic Flow vs. V/C Ratio

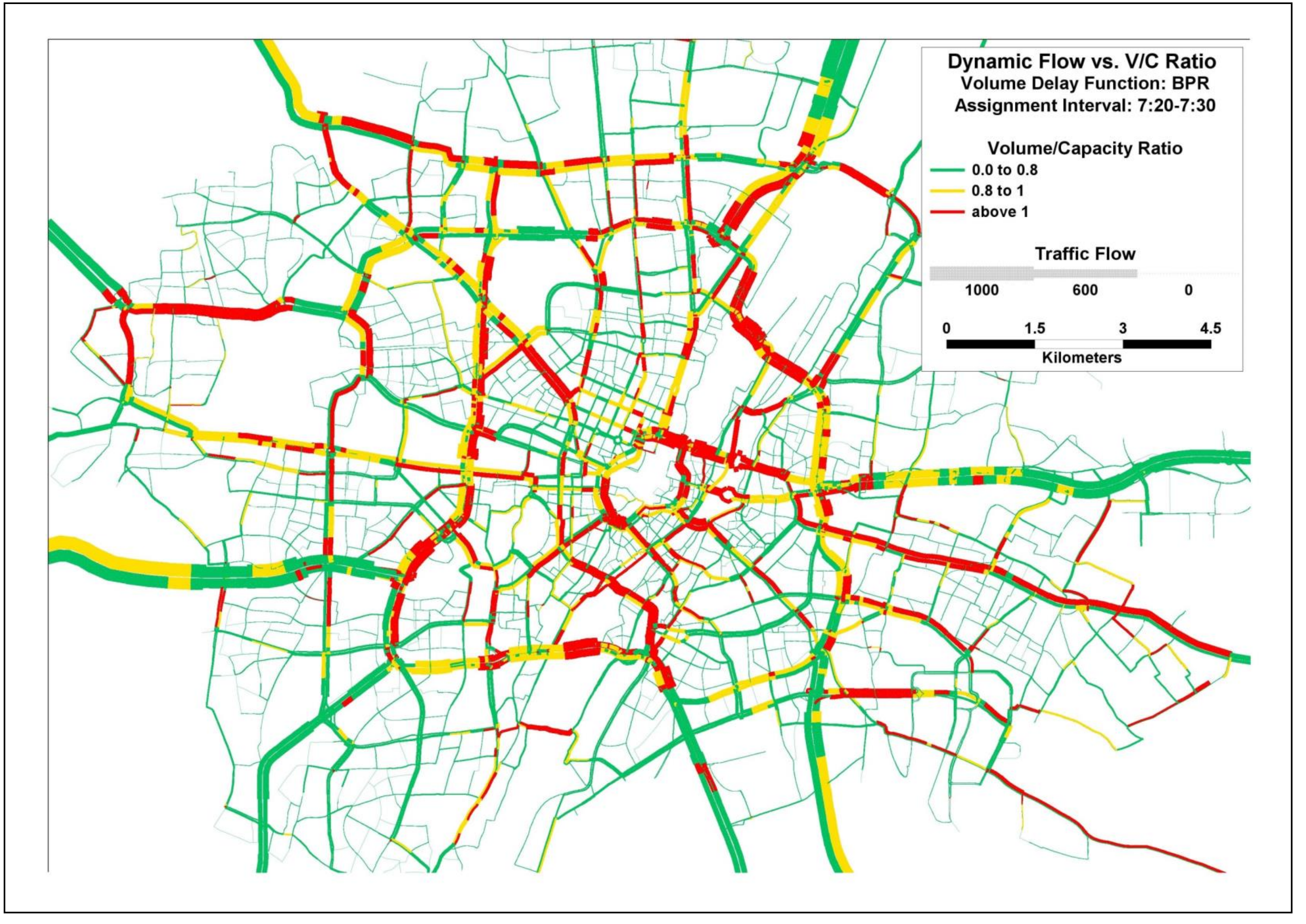
a.i) Assignment with **BPR** function for **morning** peak-hour **(7:00 – 7:10)**



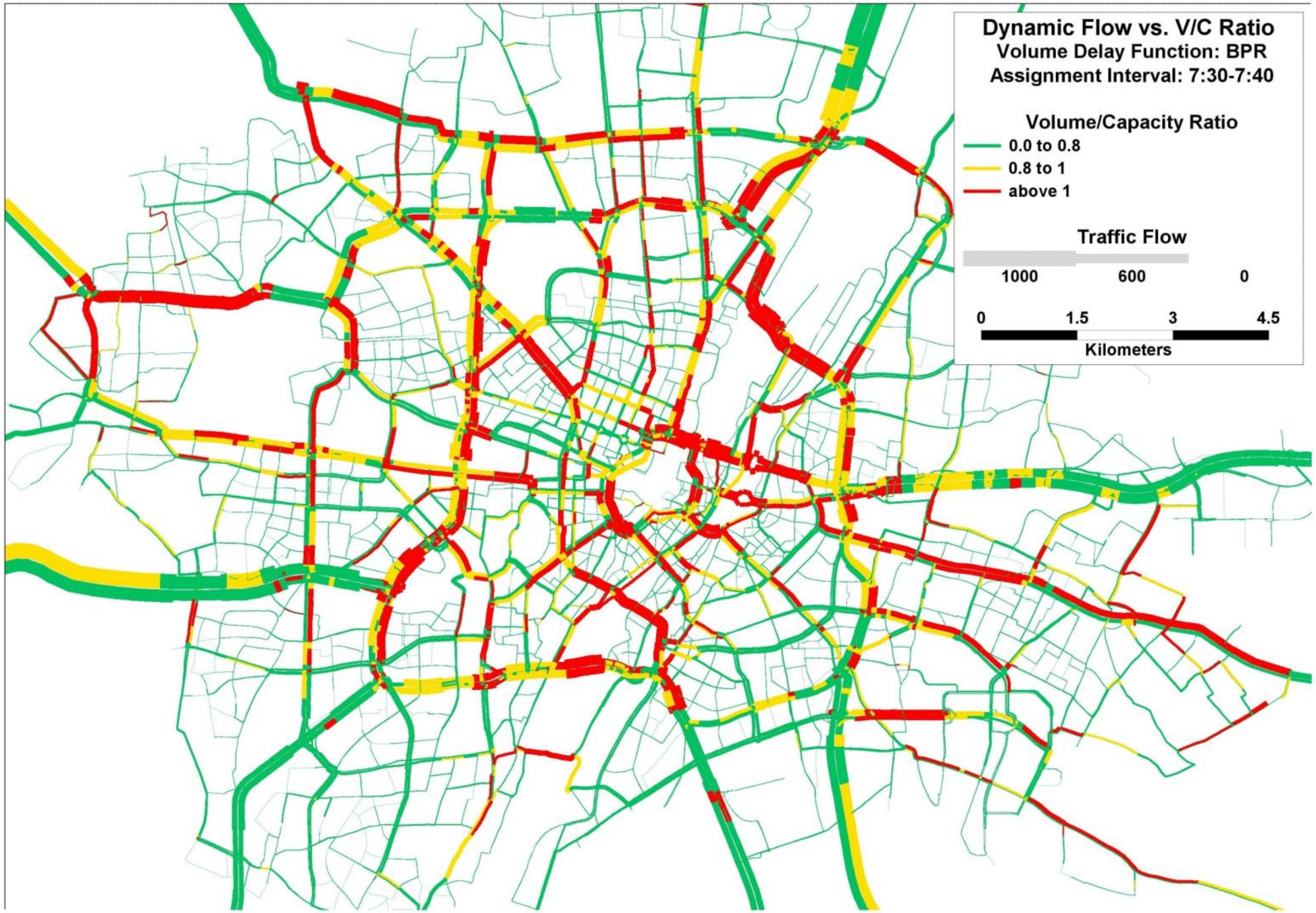
a.ii) Assignment with **BPR** function for **morning** peak-hour **(7:10 – 7:20)**



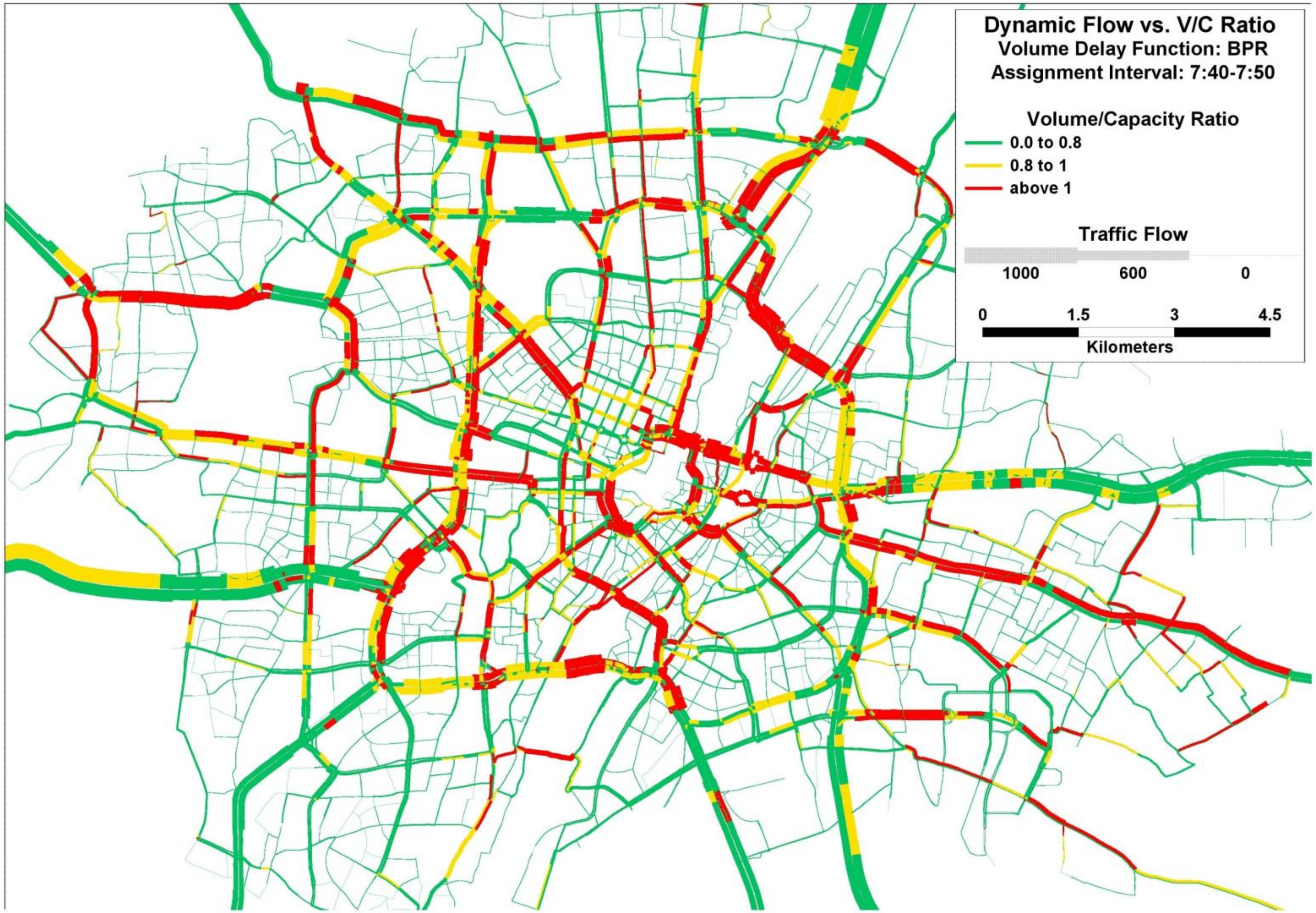
a.iii) Assignment with **BPR** function for **morning** peak-hour **(7:20 – 7:30)**



a.iv) Assignment with **BPR** function for **morning** peak-hour **(7:30 – 7:40)**



a.v) Assignment with **BPR** function for **morning** peak-hour **(7:40 – 7:50)**



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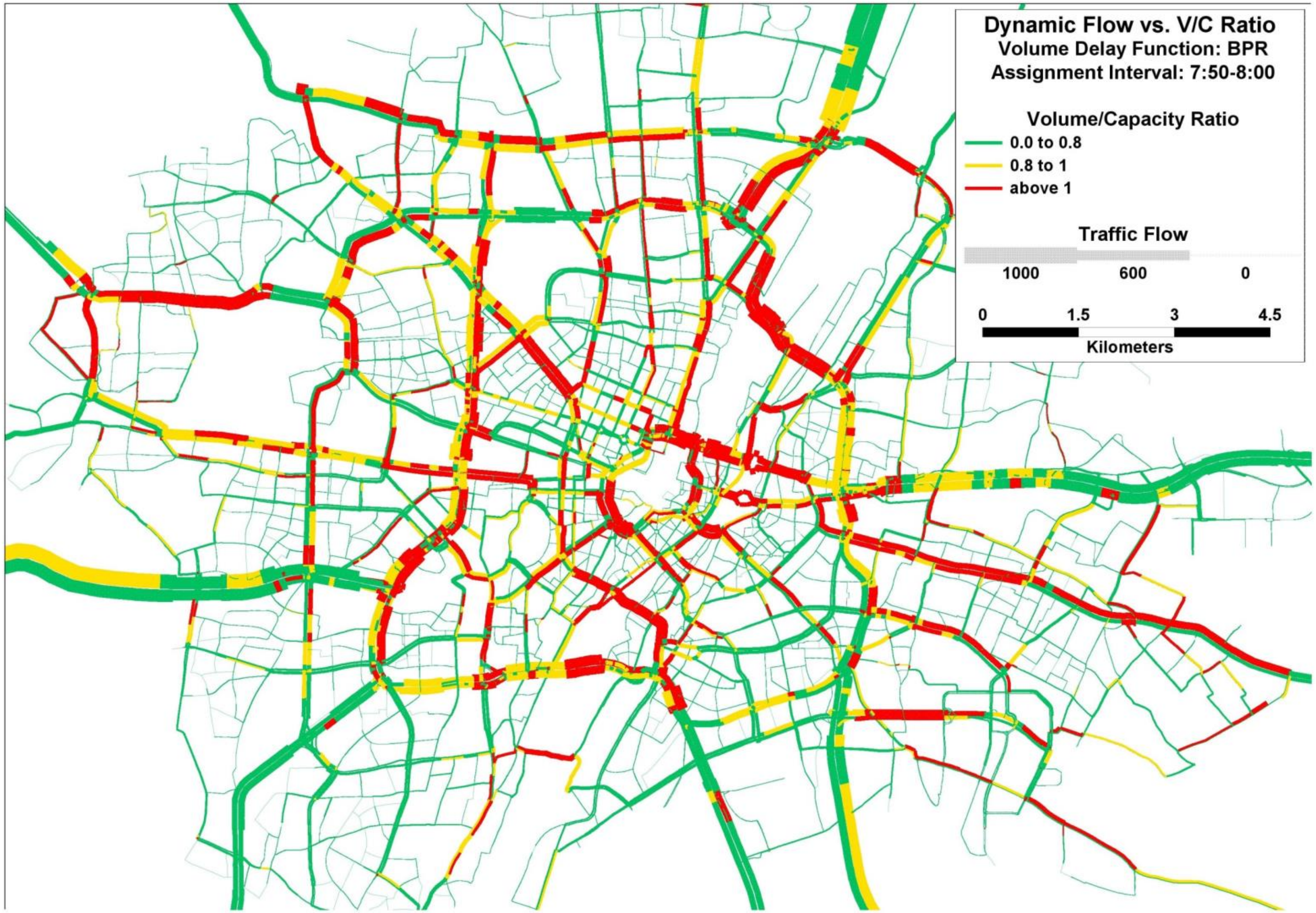
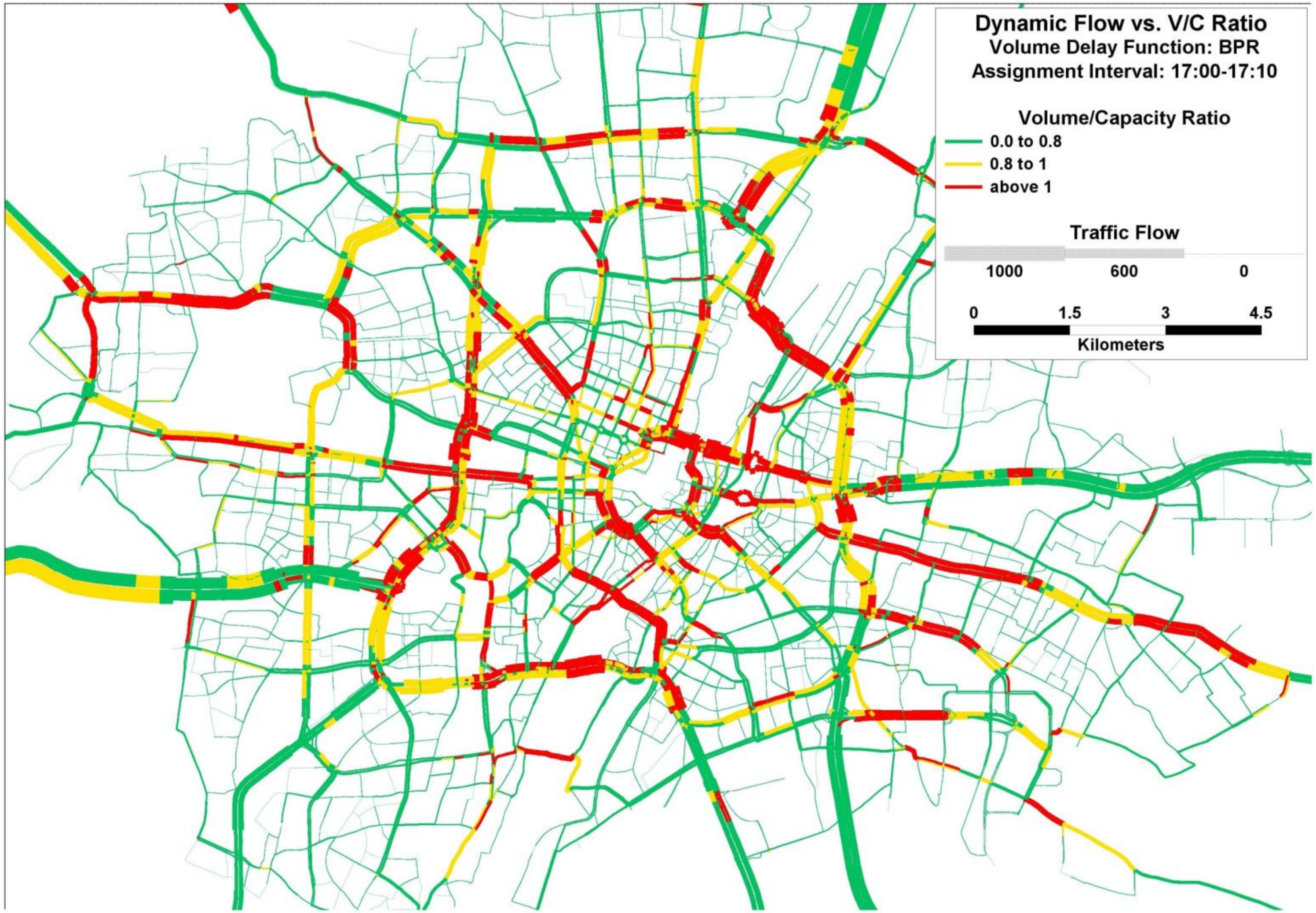
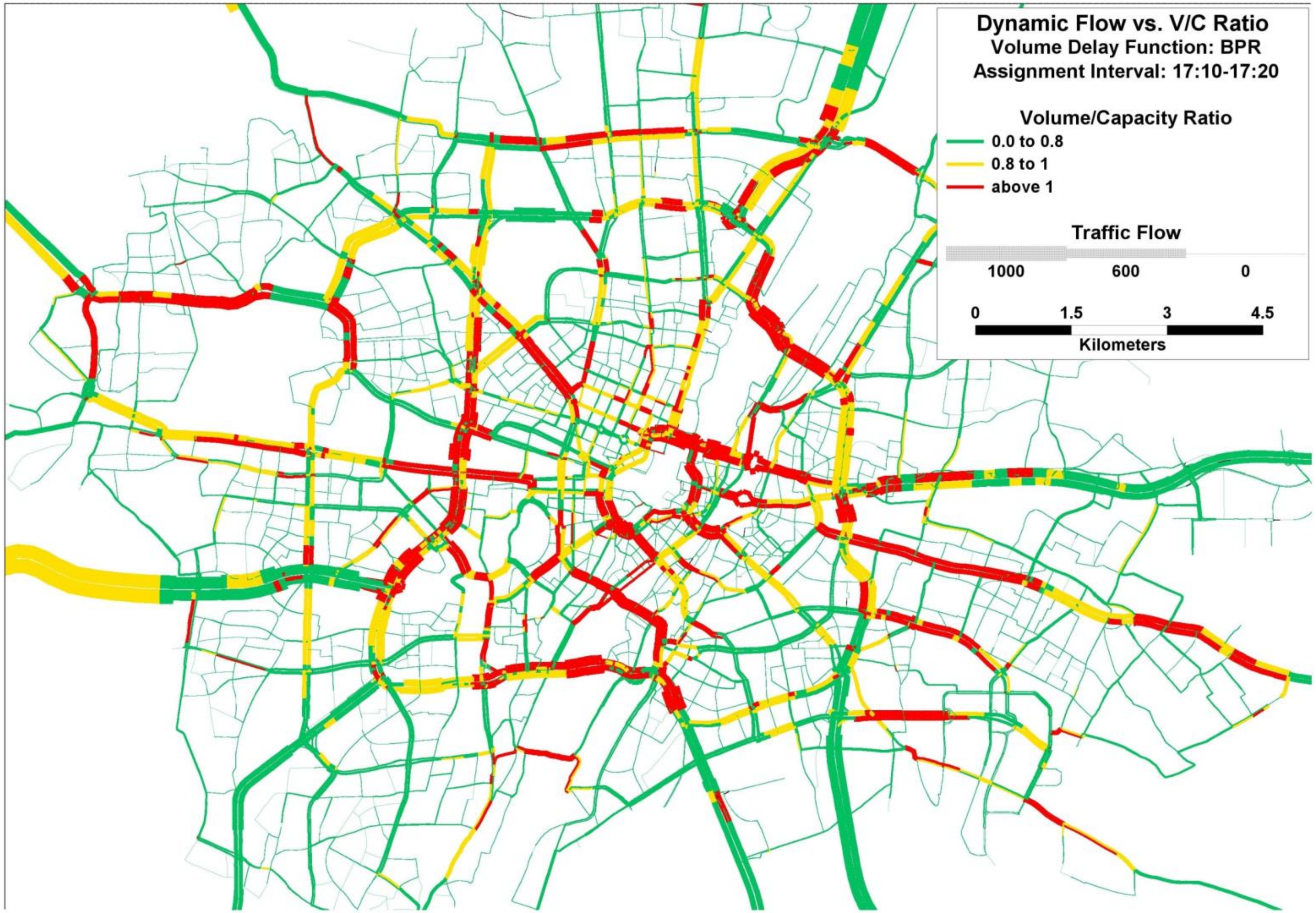


b.i) Assignment with **BPR** function for **afternoon** peak-hour **(17:00 – 17:10)**



b.ii) Assignment with **BPR** function for **afternoon** peak-hour **(17:10 – 17:20)**



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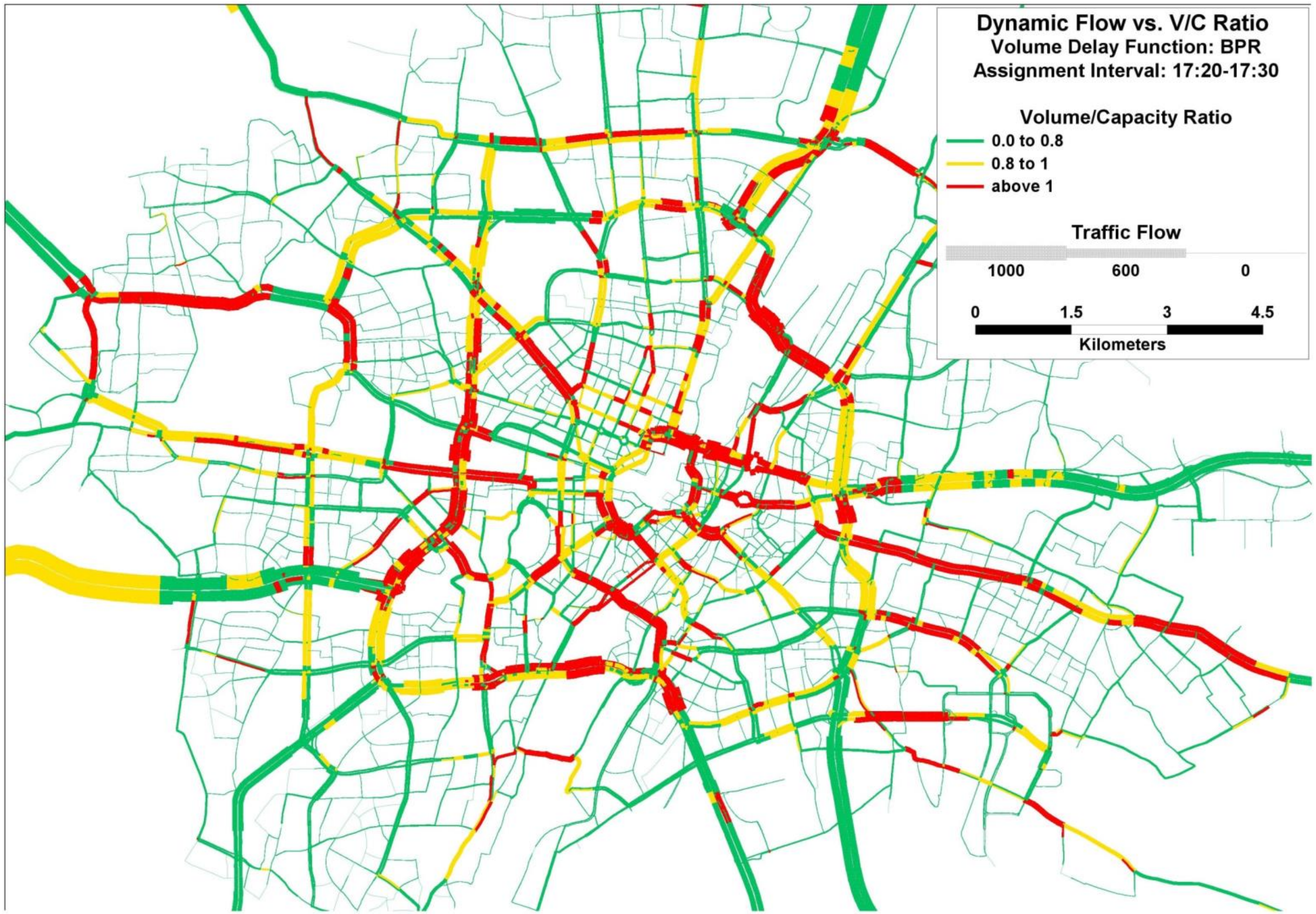
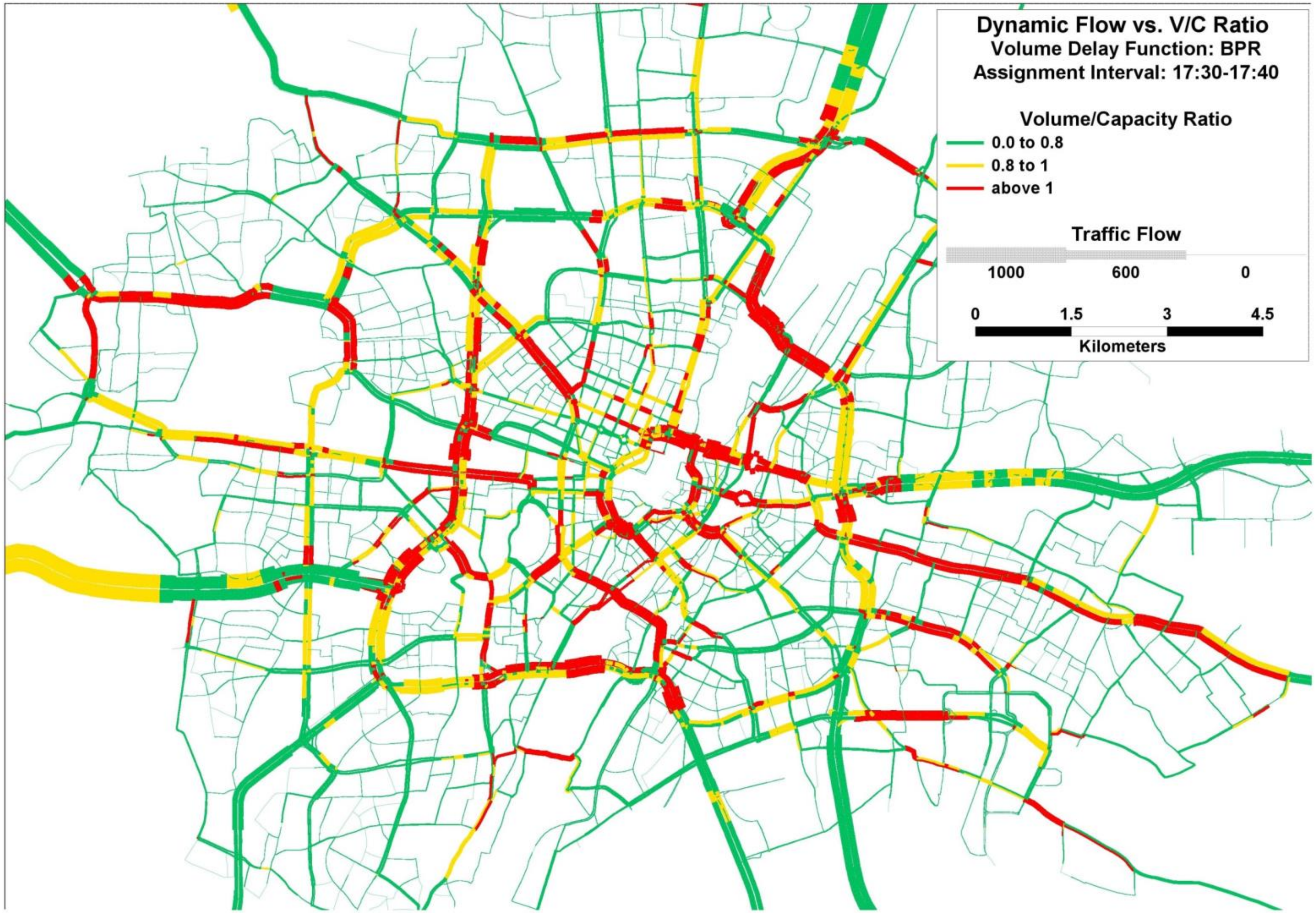


b.iv) Assignment with **BPR** function for **afternoon** peak-hour **(17:30 – 17:40)**



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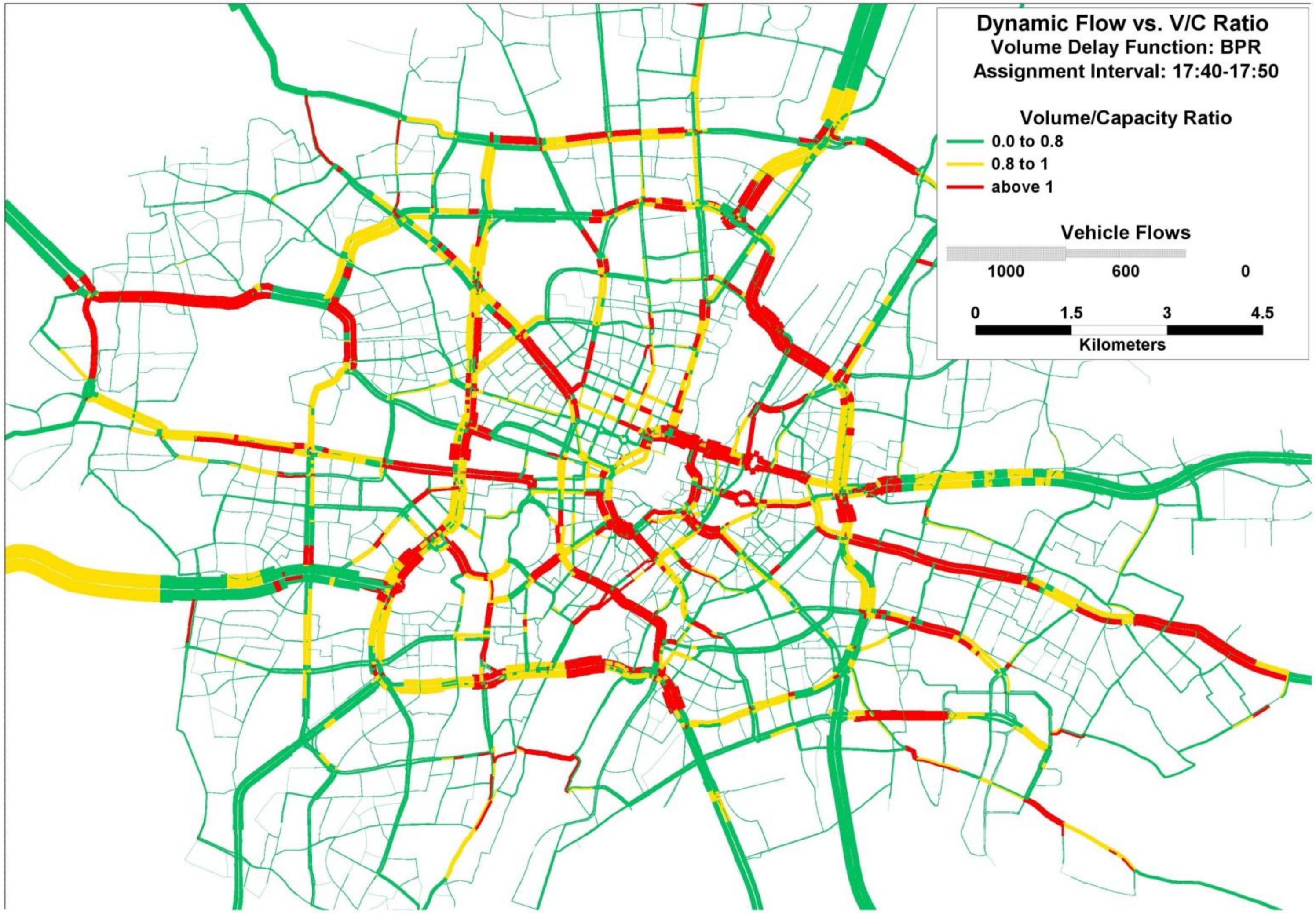
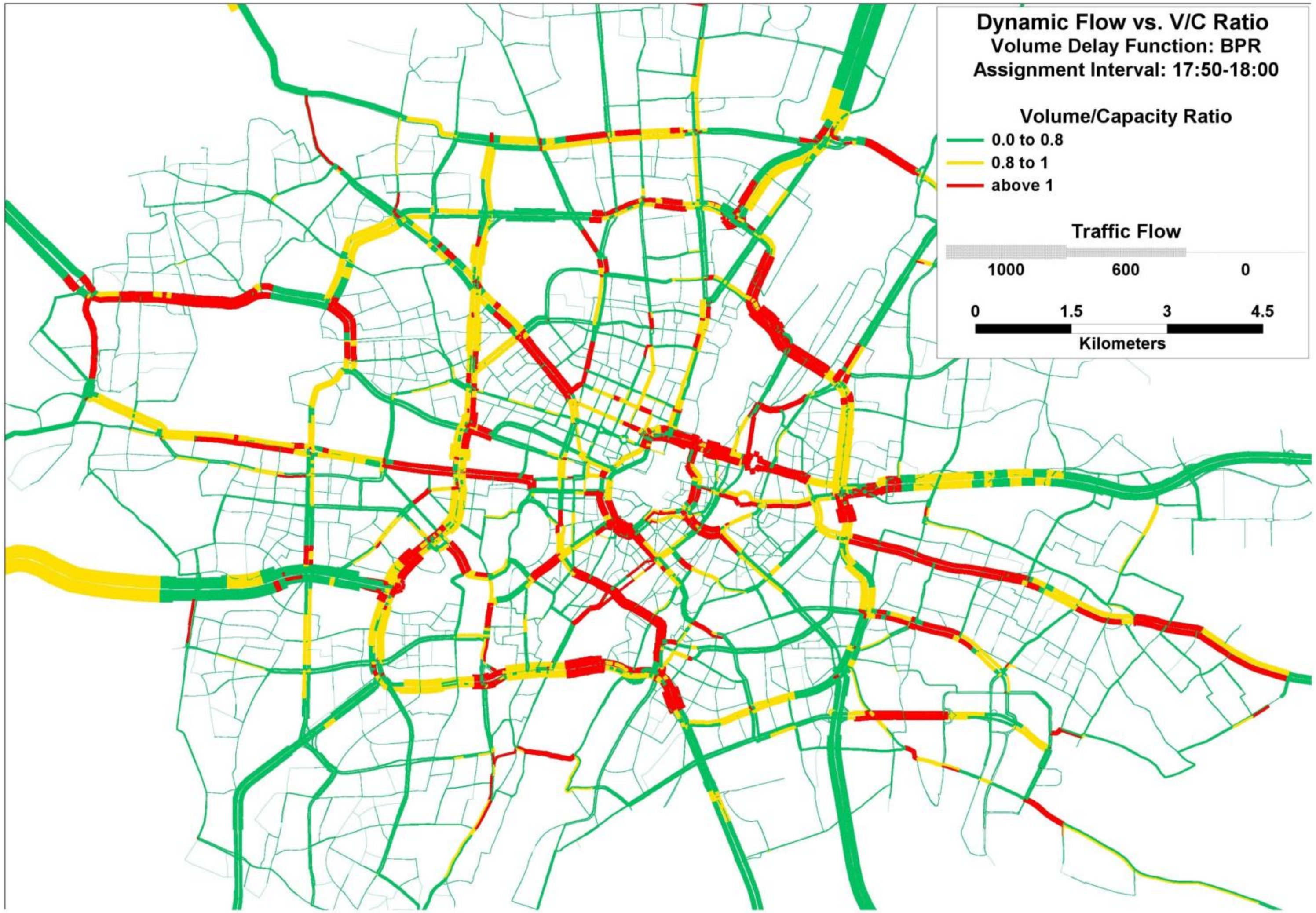
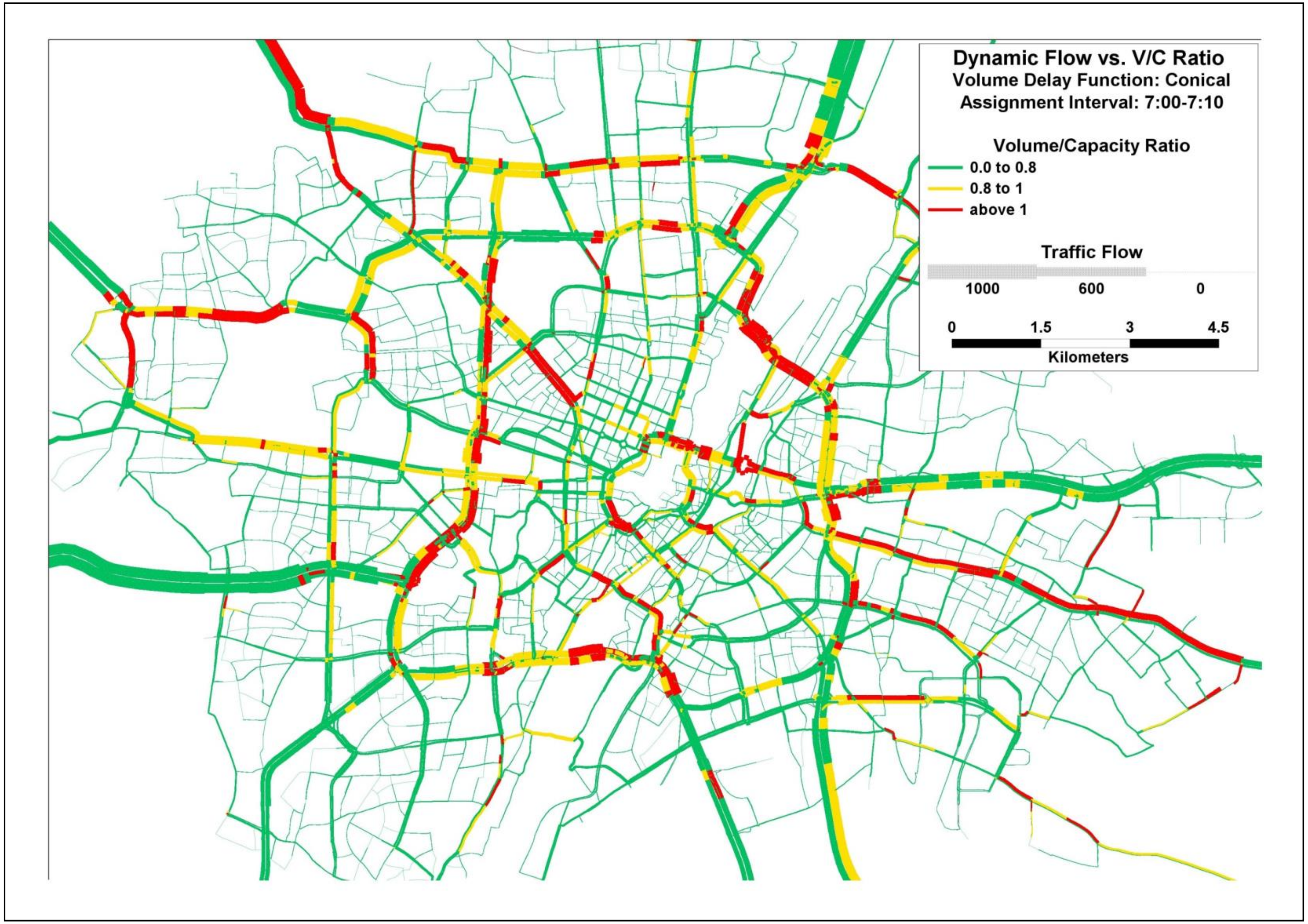


b.vi) Assignment with **BPR** function for **afternoon** peak-hour **(17:50 – 18:00)**



c.i) Assignment with **Conical** function for **morning** peak-hour **(7:00 – 7:10)**



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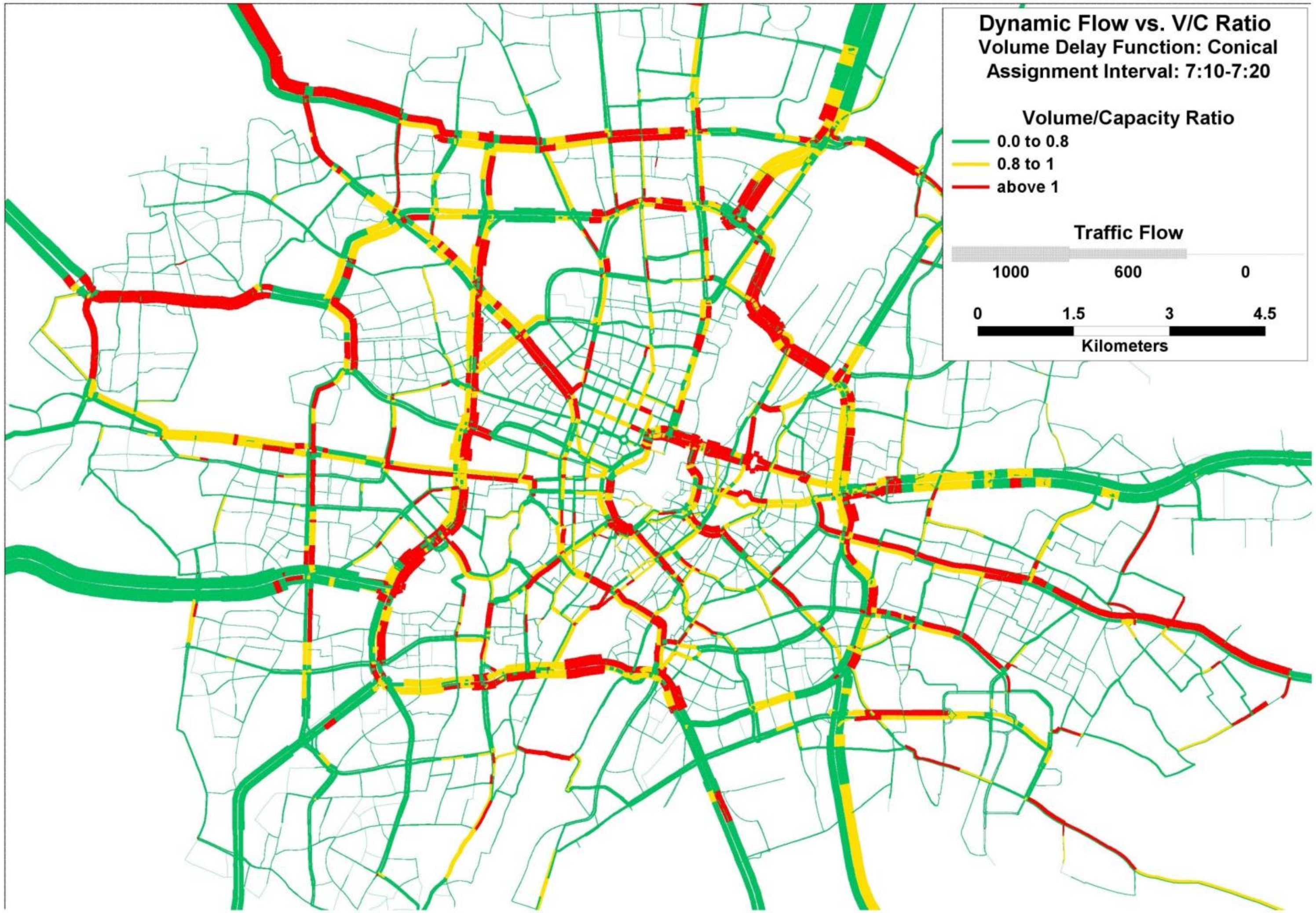

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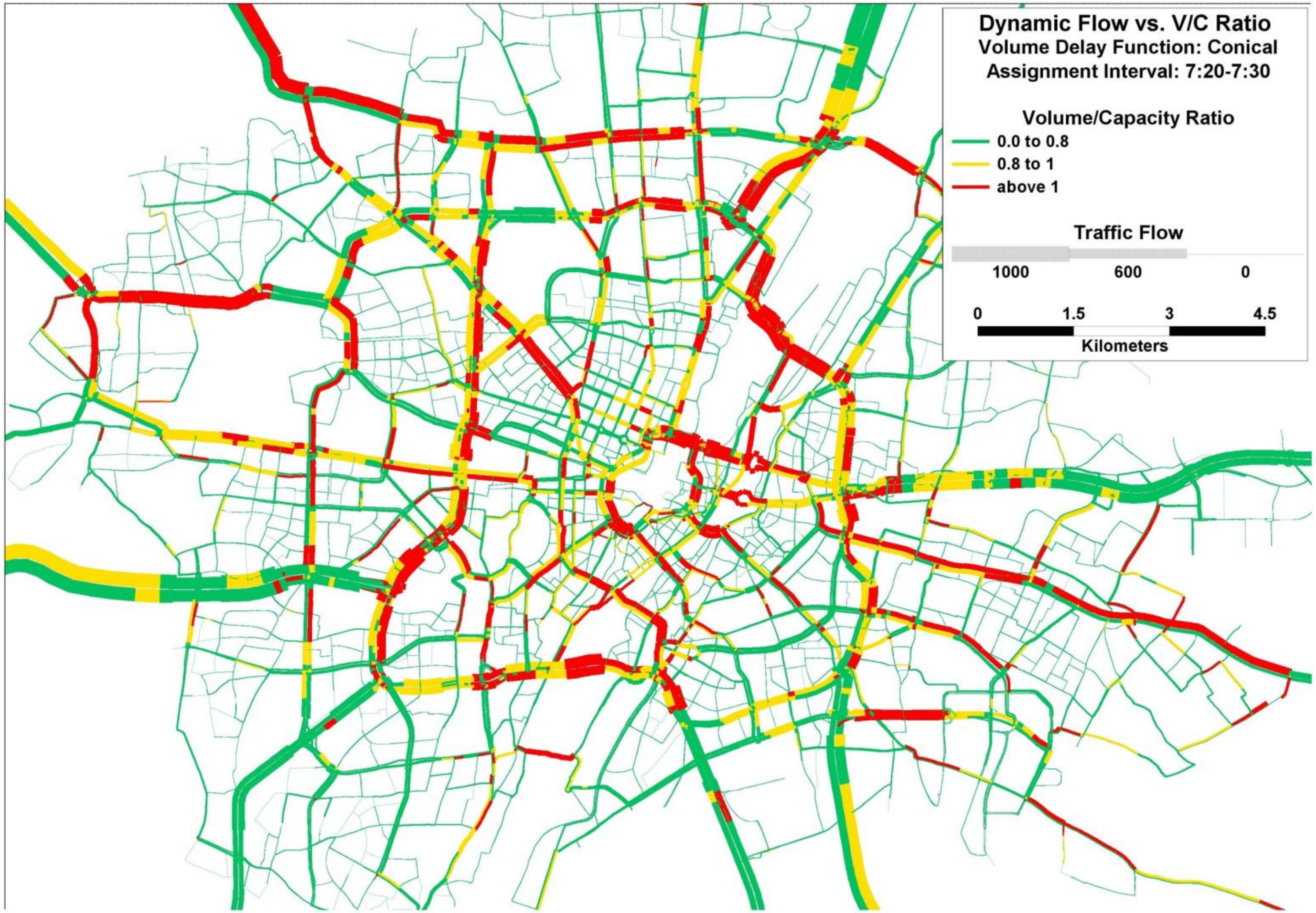

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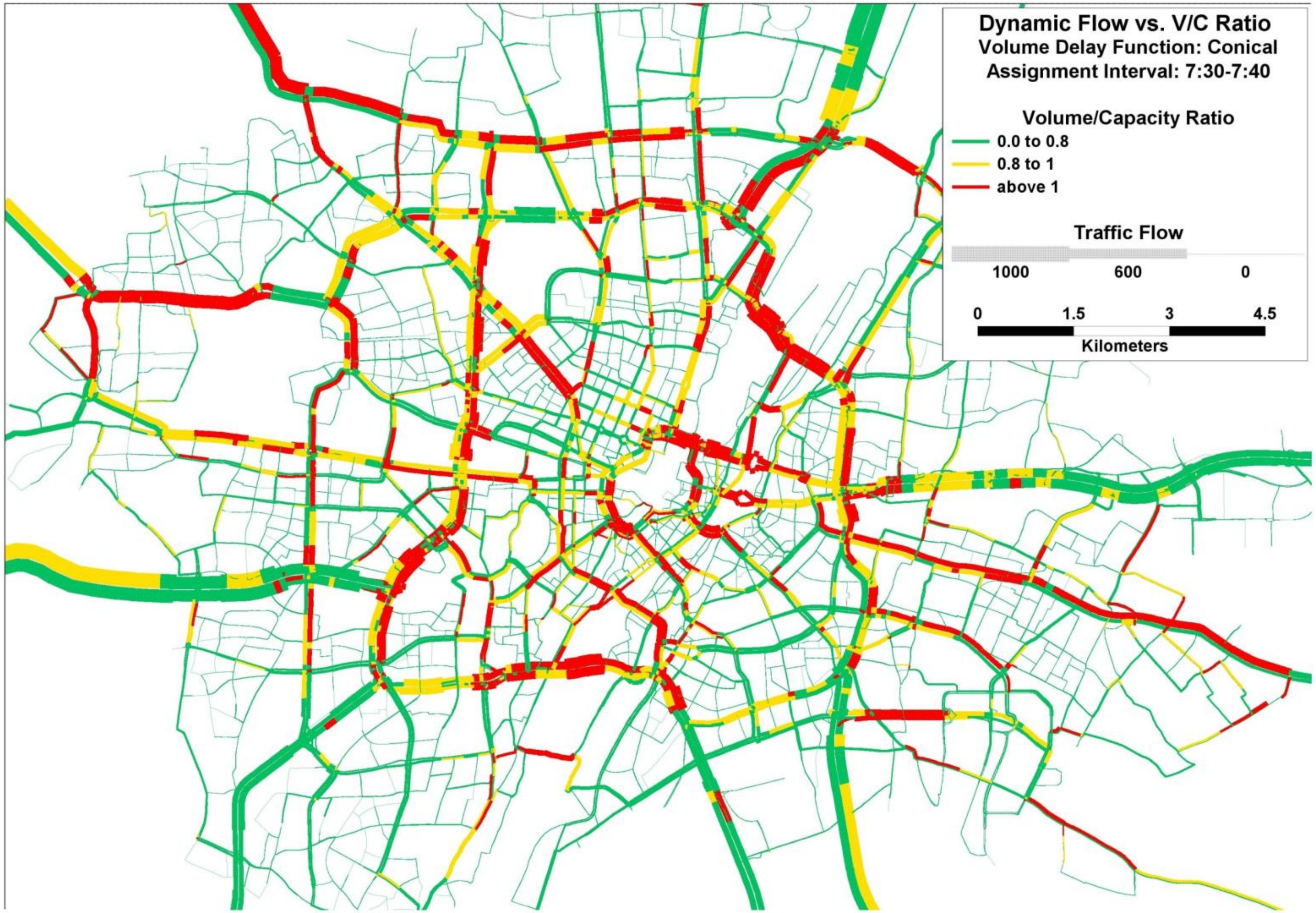
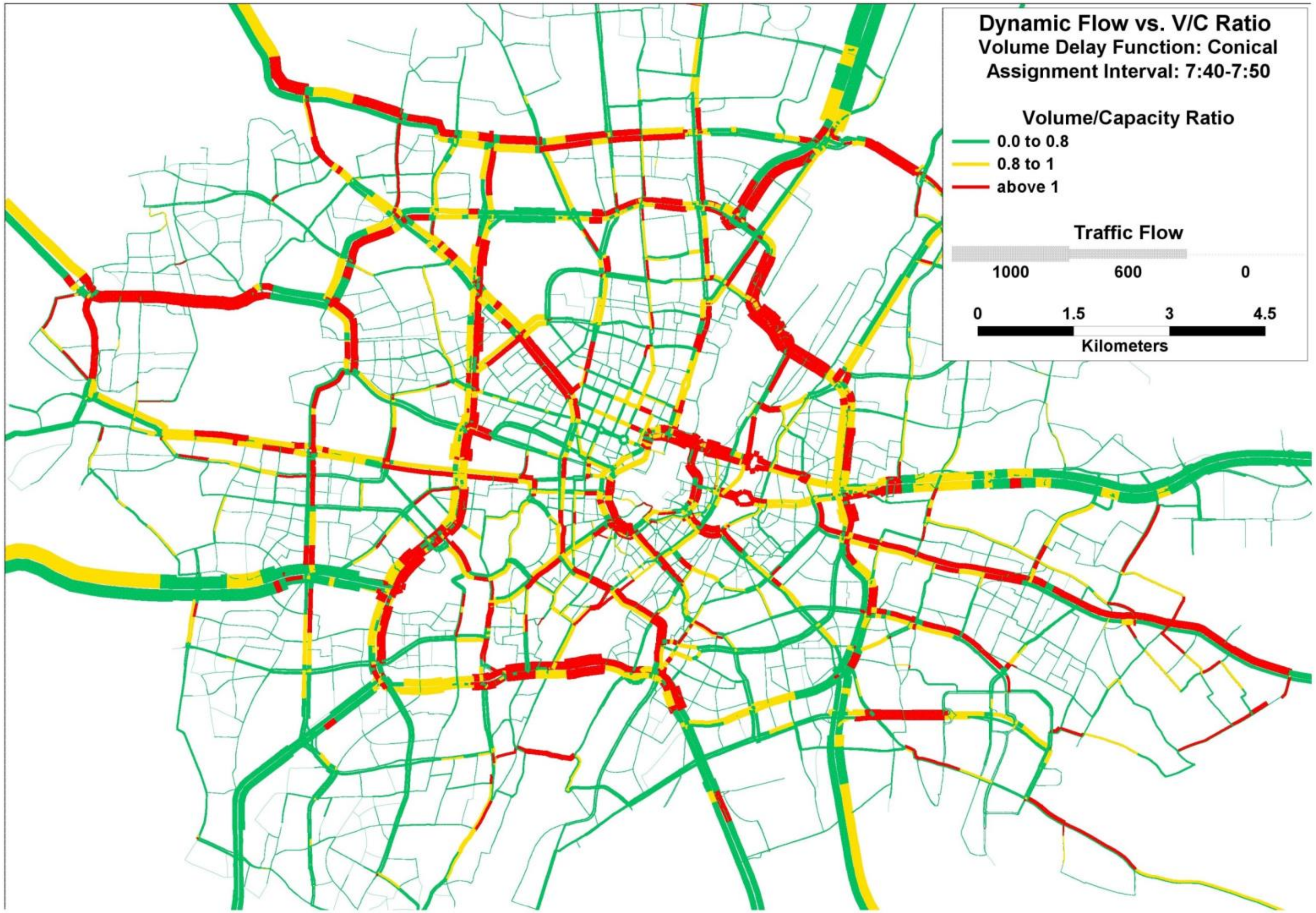


c.v) Assignment with **Conical** function for **morning** peak-hour **(7:40 – 7:50)**



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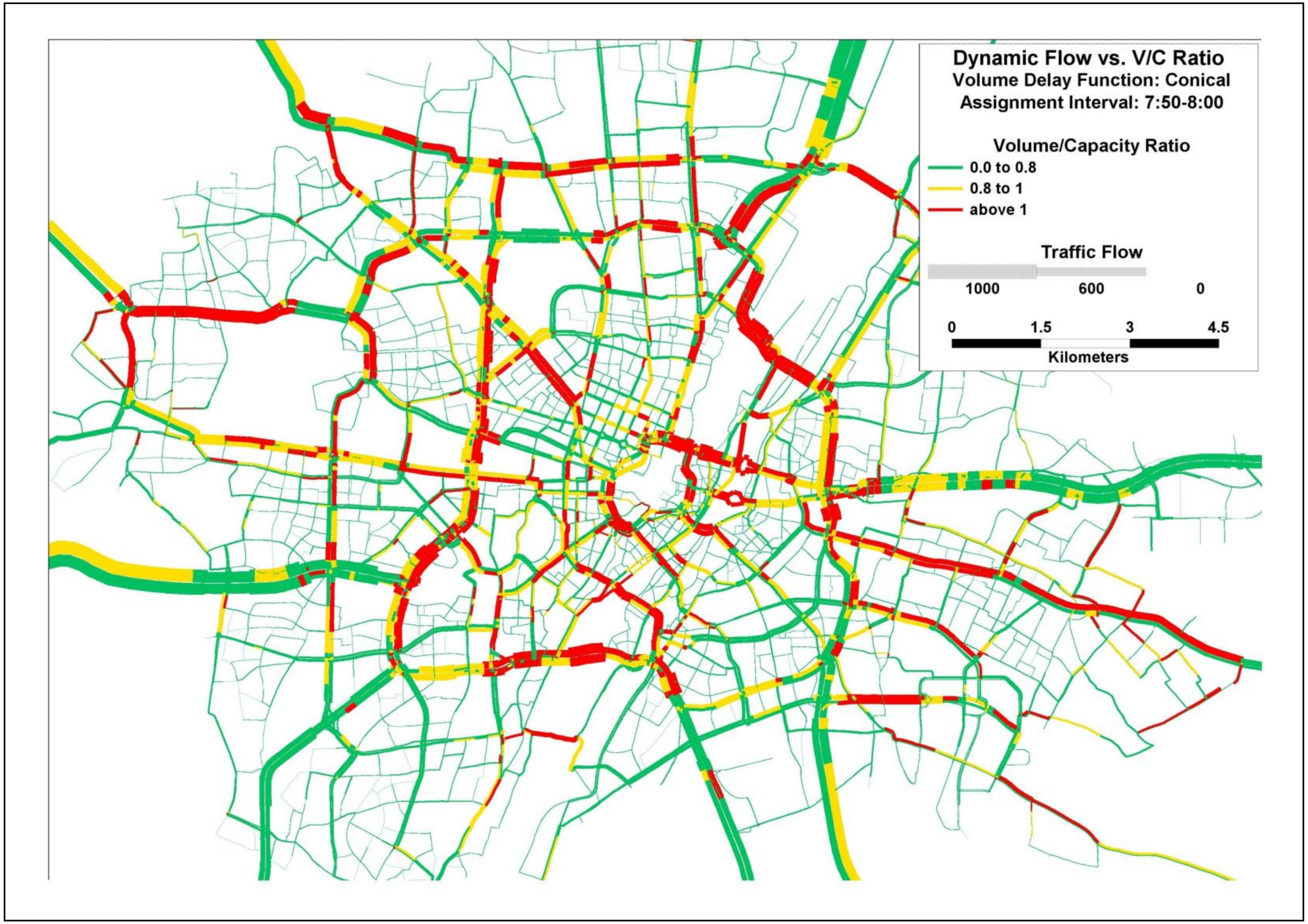
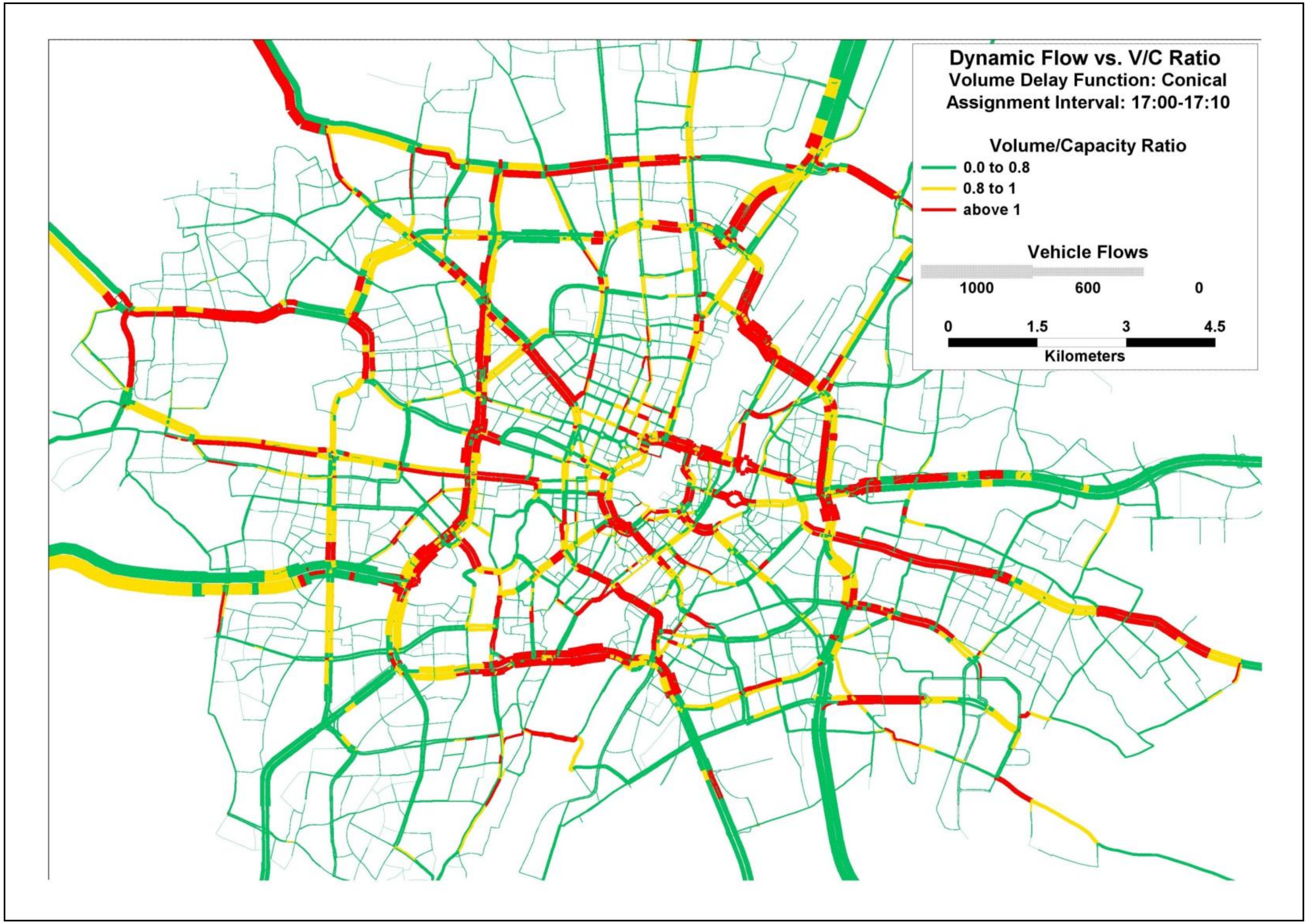
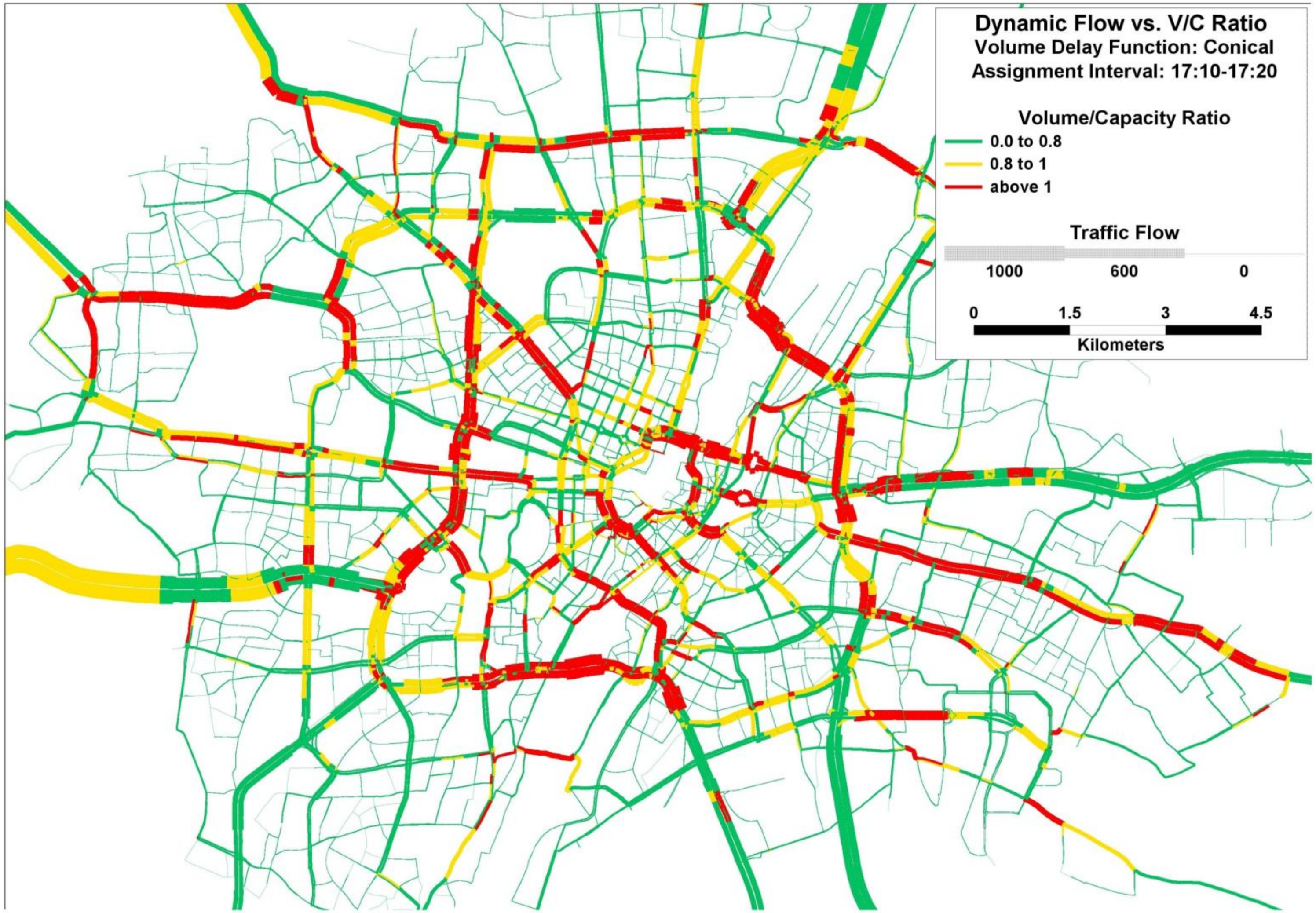


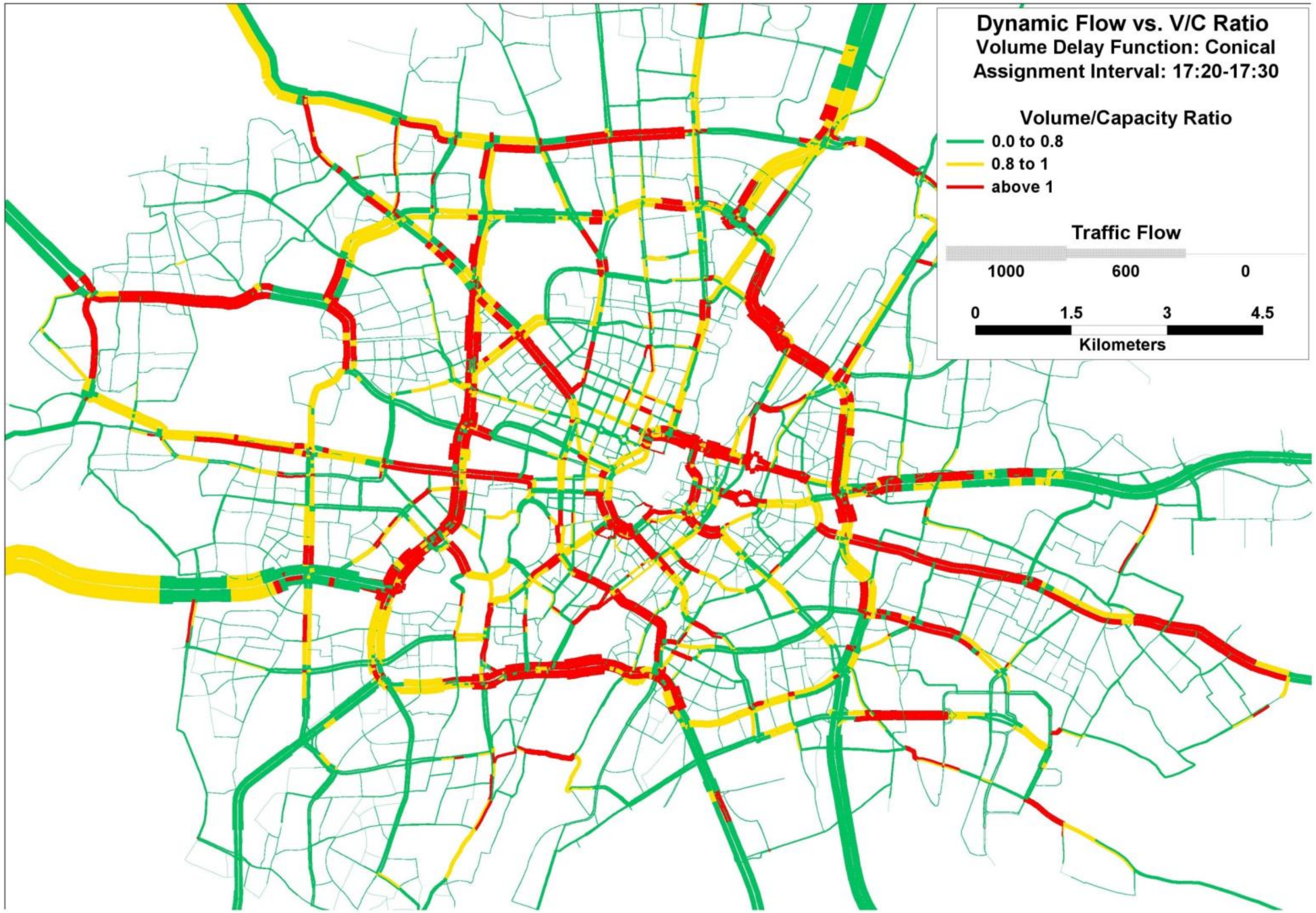
d.i) Assignment with **Conical** function for **afternoon** peak-hour **(17:00 – 17:10)**



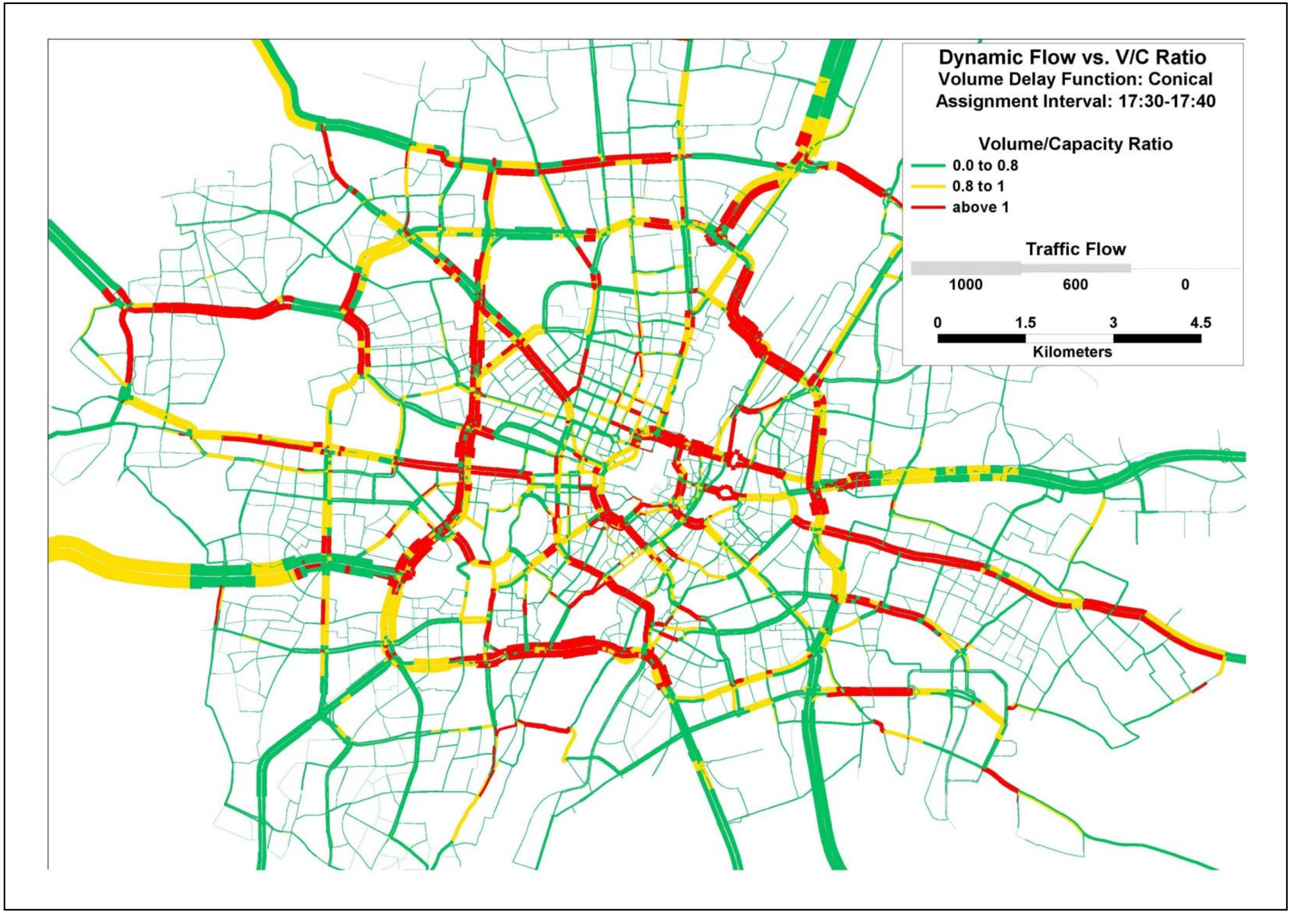
d.ii) Assignment with **Conical** function for **afternoon** peak-hour **(17:10 – 17:20)**



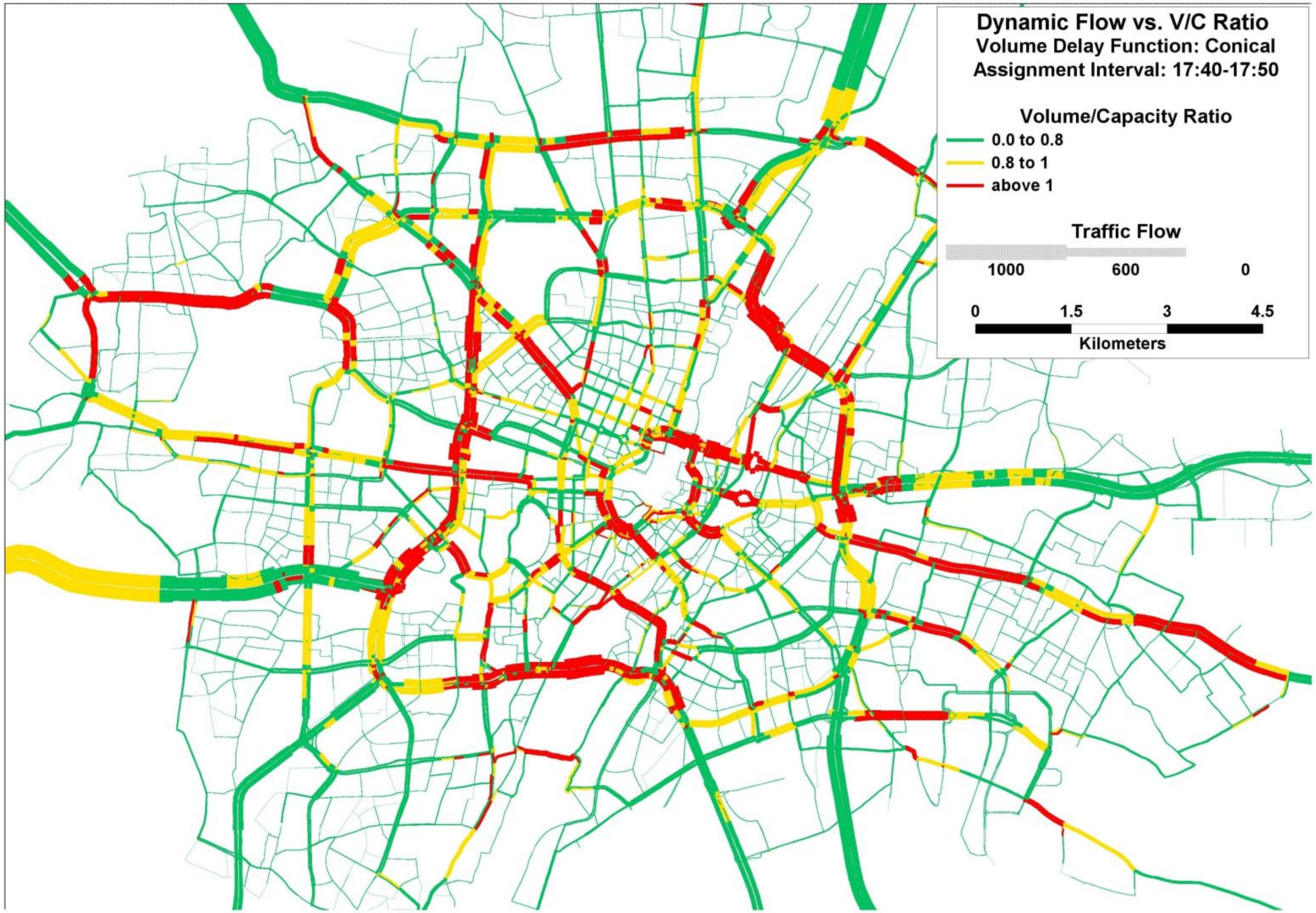
d.iii) Assignment with **Conical** function for **afternoon** peak-hour **(17:20 – 17:30)**



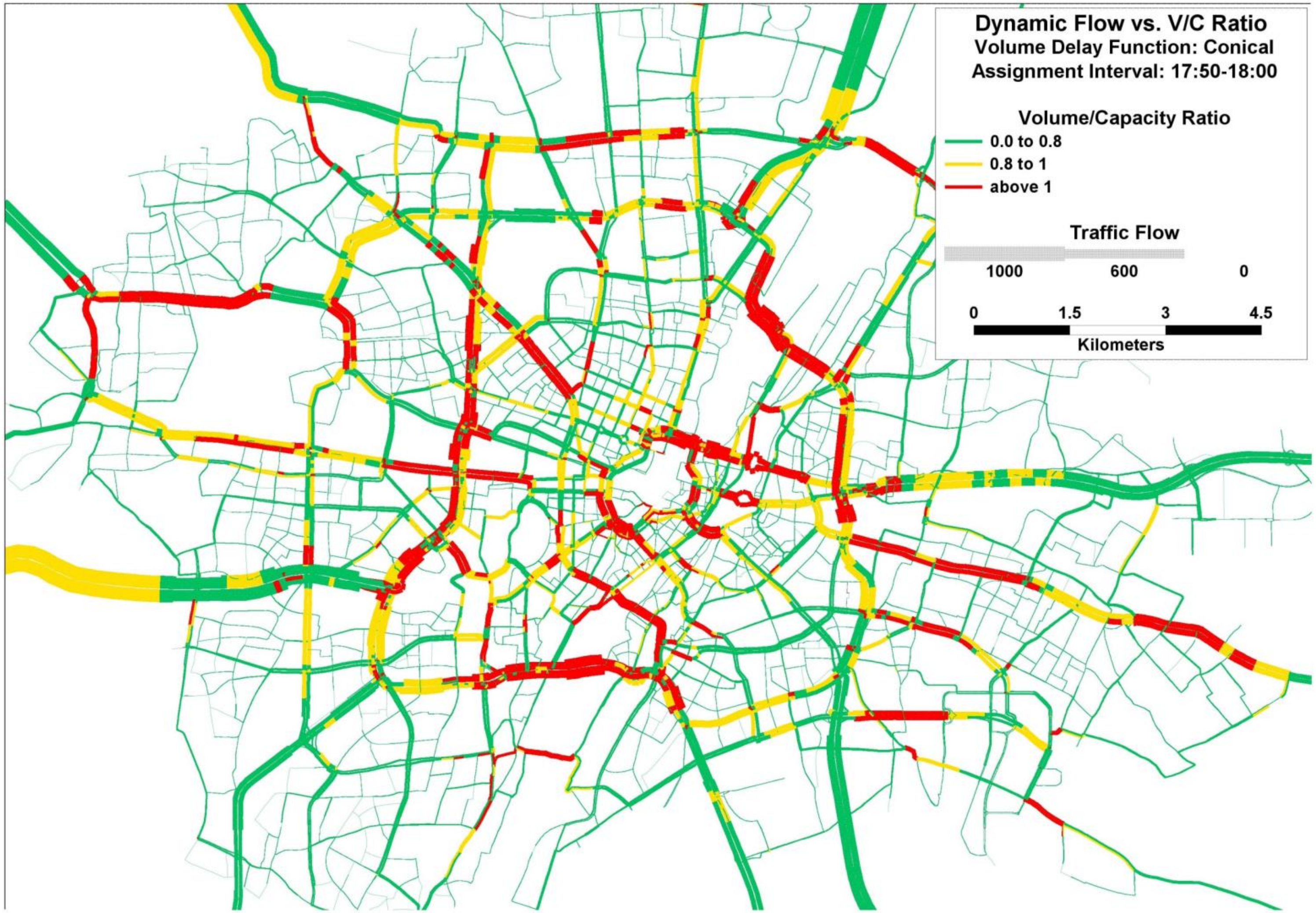
d.iv) Assignment with **Conical** function for **afternoon** peak-hour **(17:30 – 17:40)**



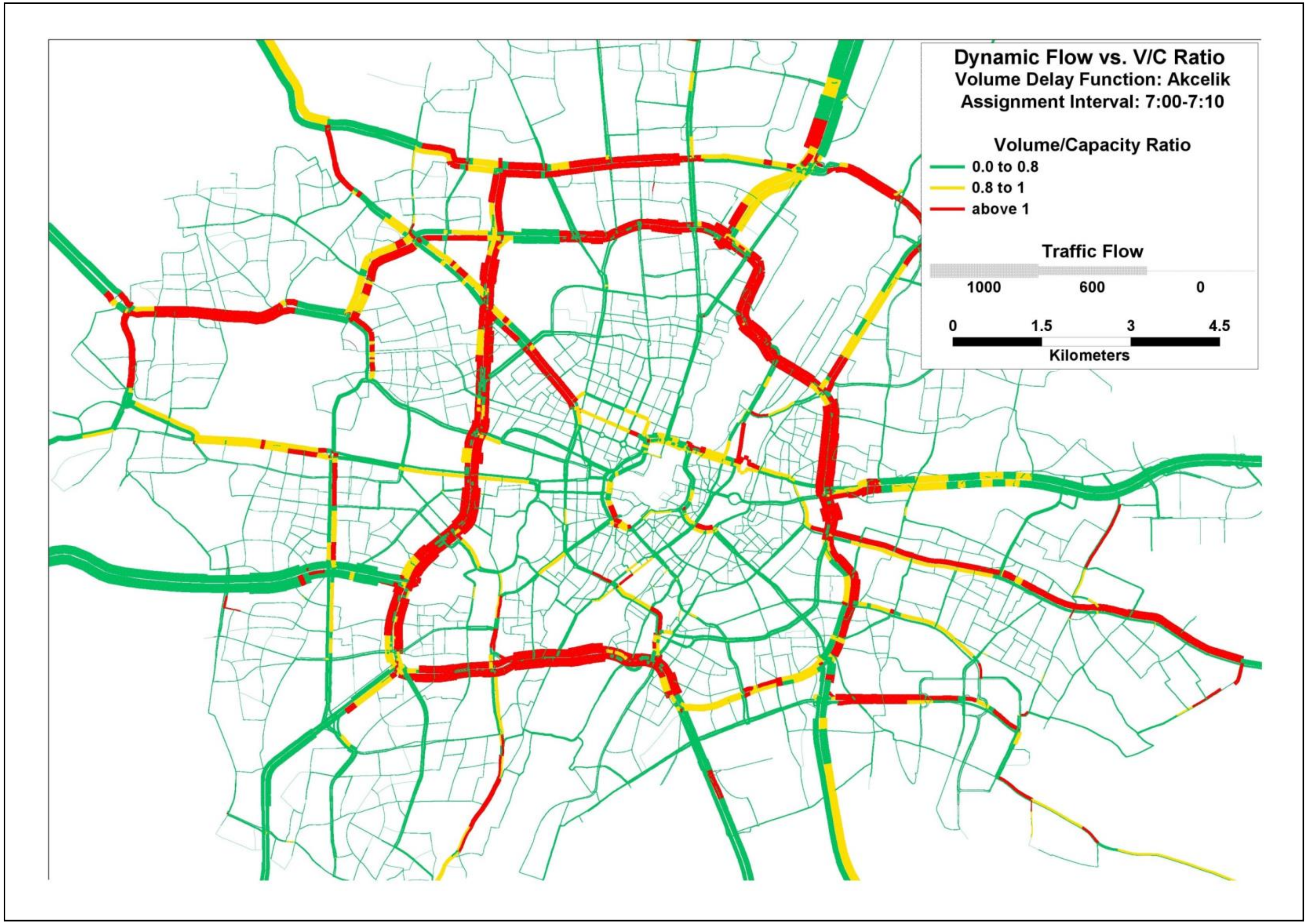
d.v) Assignment with **Conical** function for **afternoon** peak-hour **(17:40 – 17:50)**



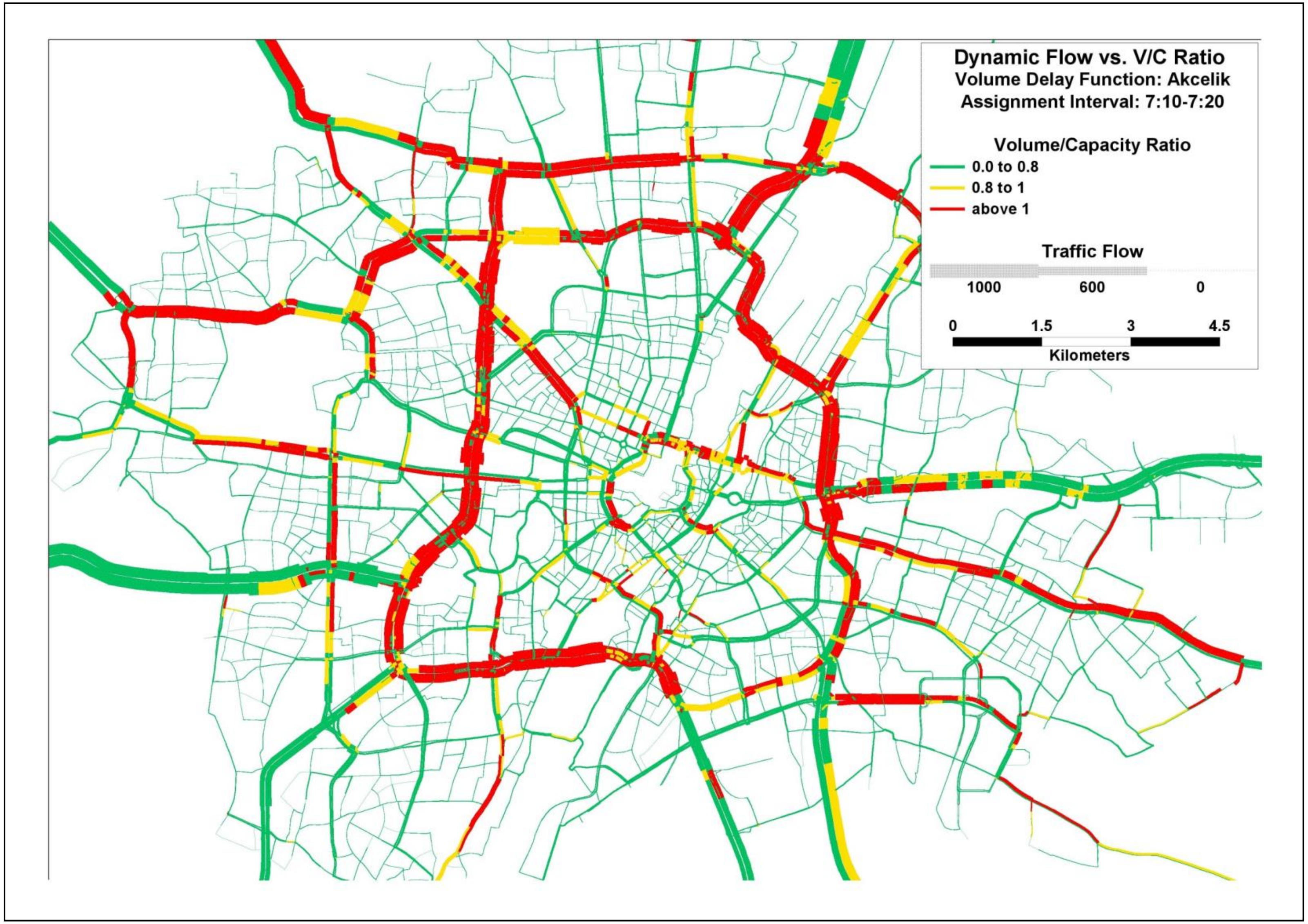
d.vi) Assignment with **Conical** function for **afternoon** peak-hour **(17:50 – 18:00)**



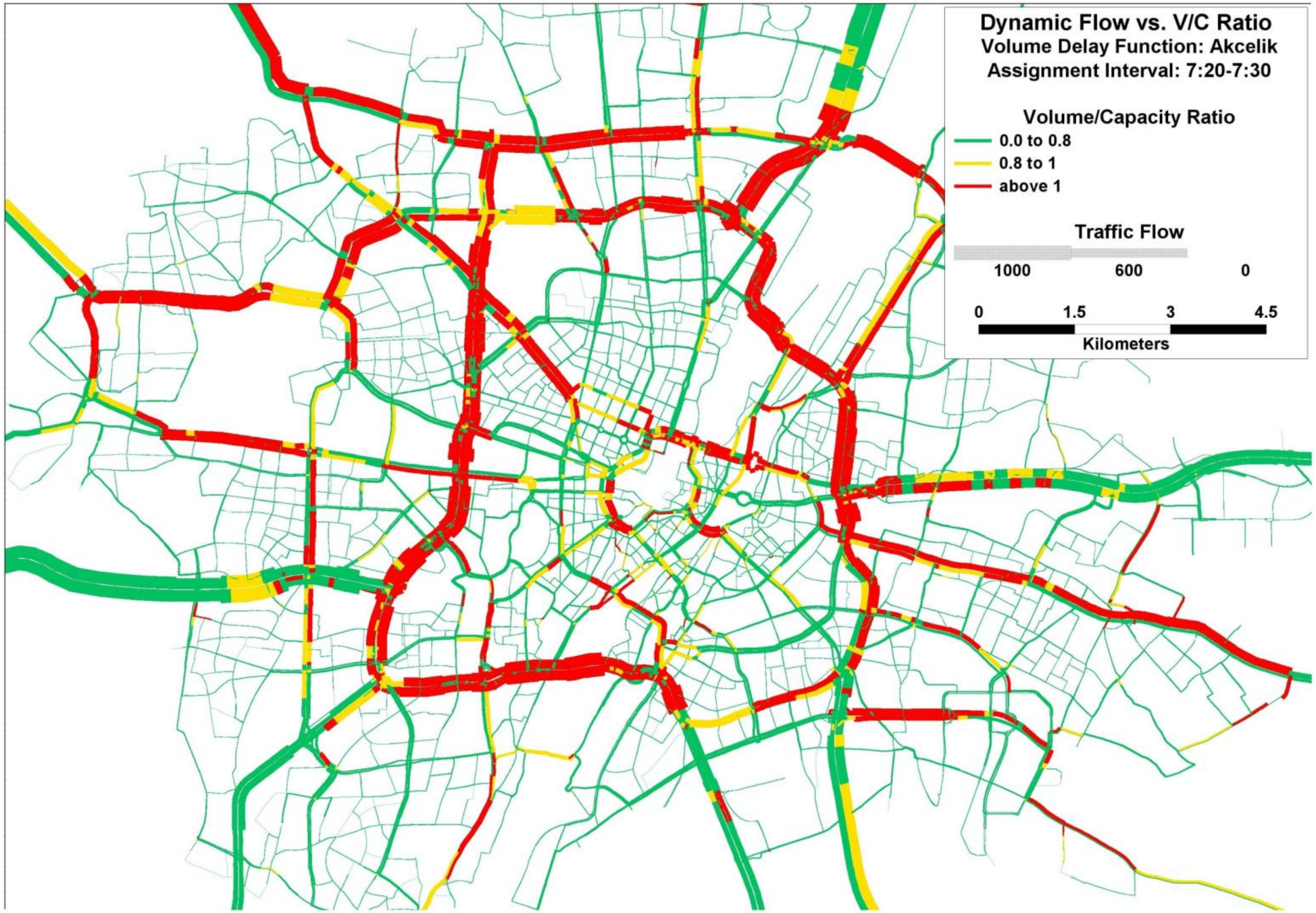
e.i) Assignment with **Akcelik** function for **morning** peak-hour **(7:00 – 7:10)**



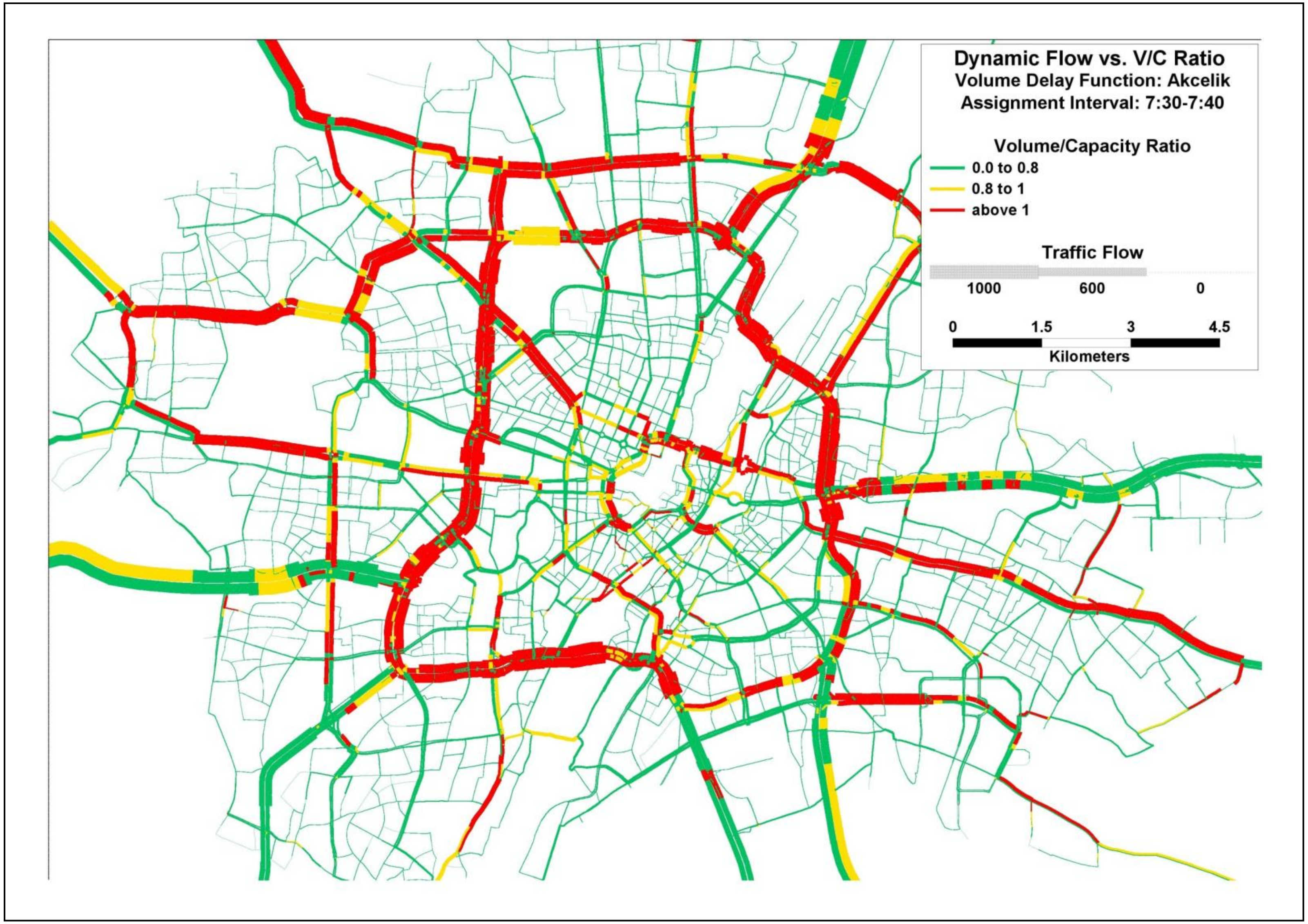
e.ii) Assignment with **Akcelik** function for **morning** peak-hour **(7:10 – 7:20)**



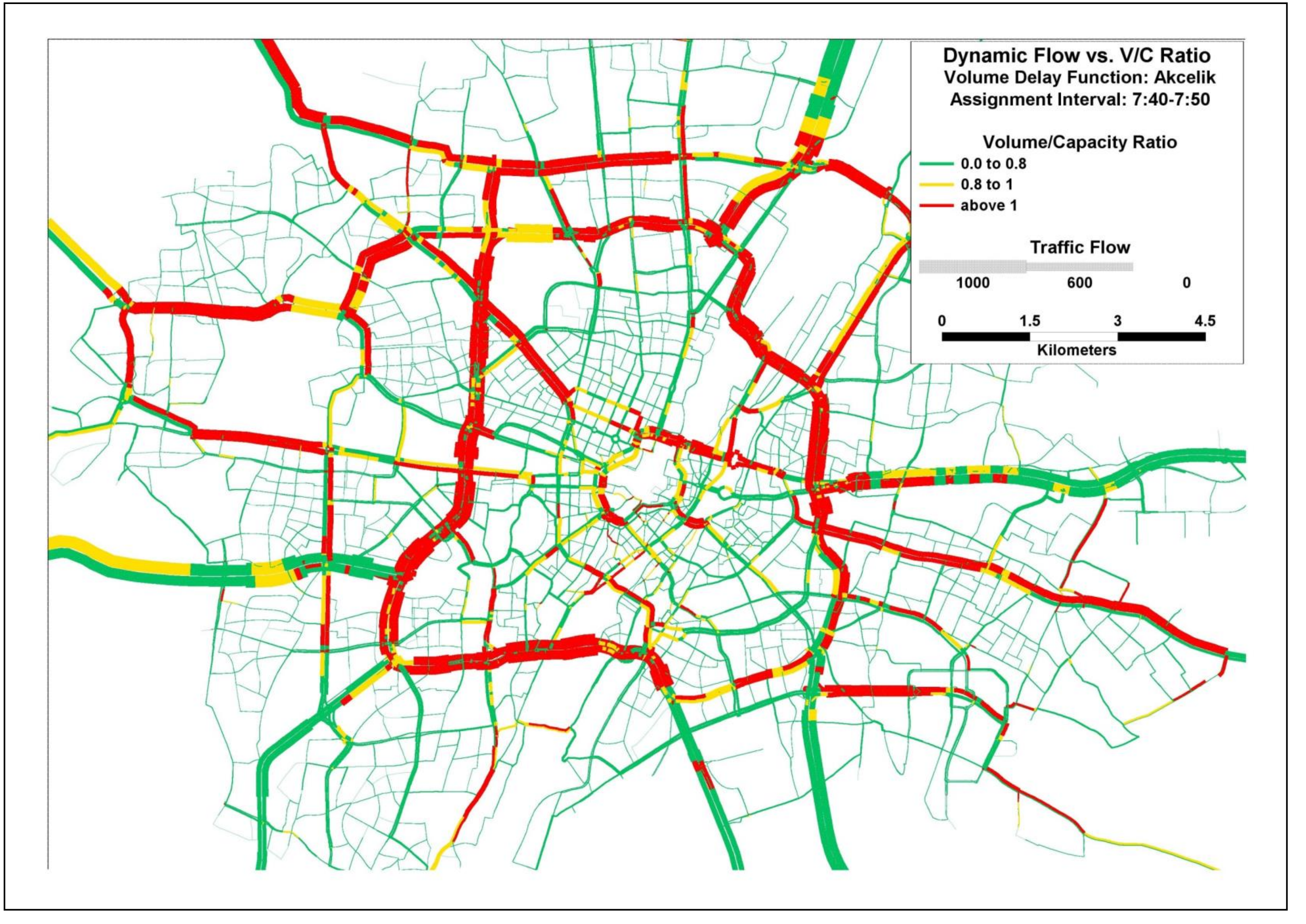
e.iii) Assignment with **Akcelik** function for **morning** peak-hour **(7:20 – 7:30)**



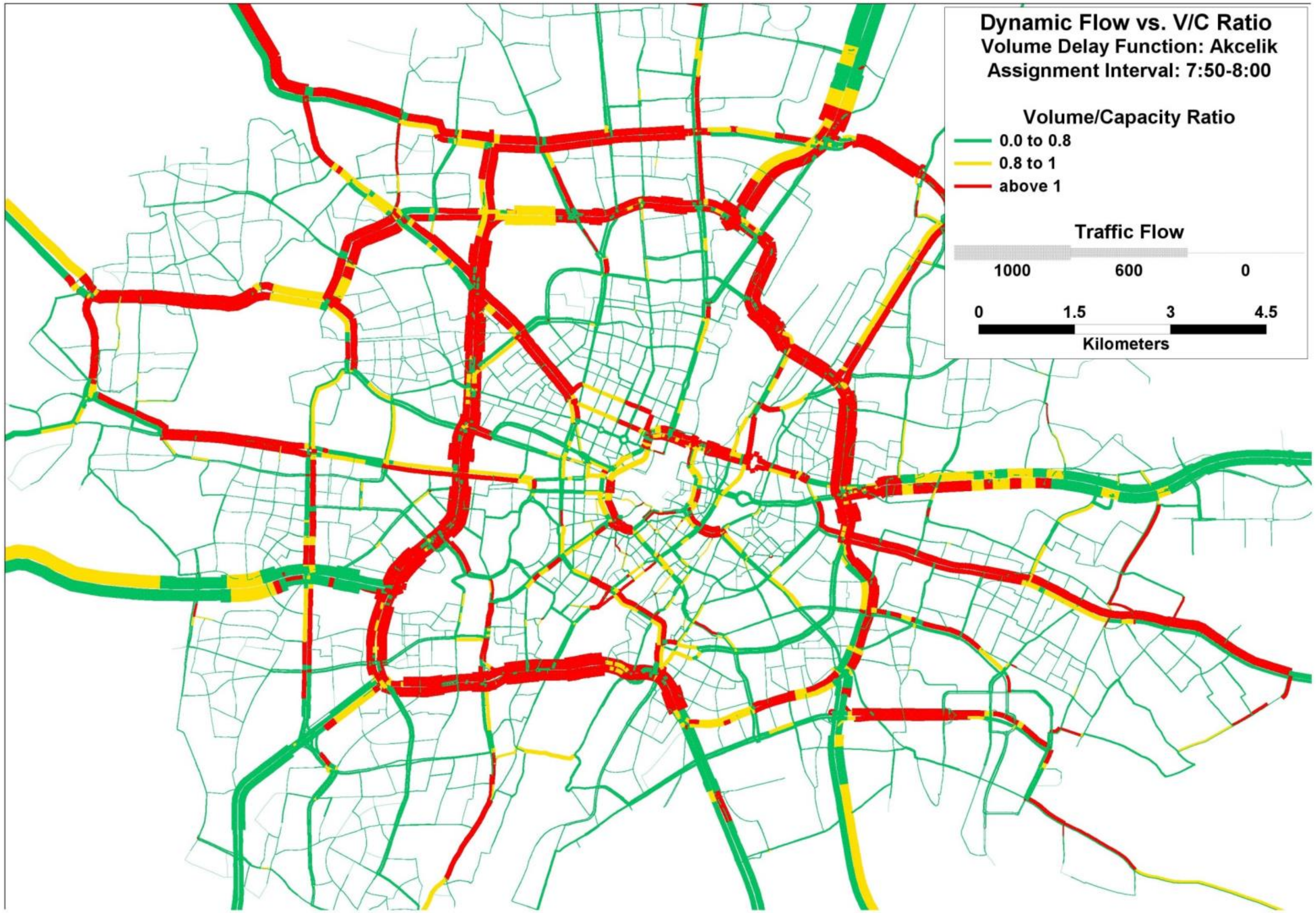
e.iv) Assignment with **Akcelik** function for **morning** peak-hour **(7:30 – 7:40)**



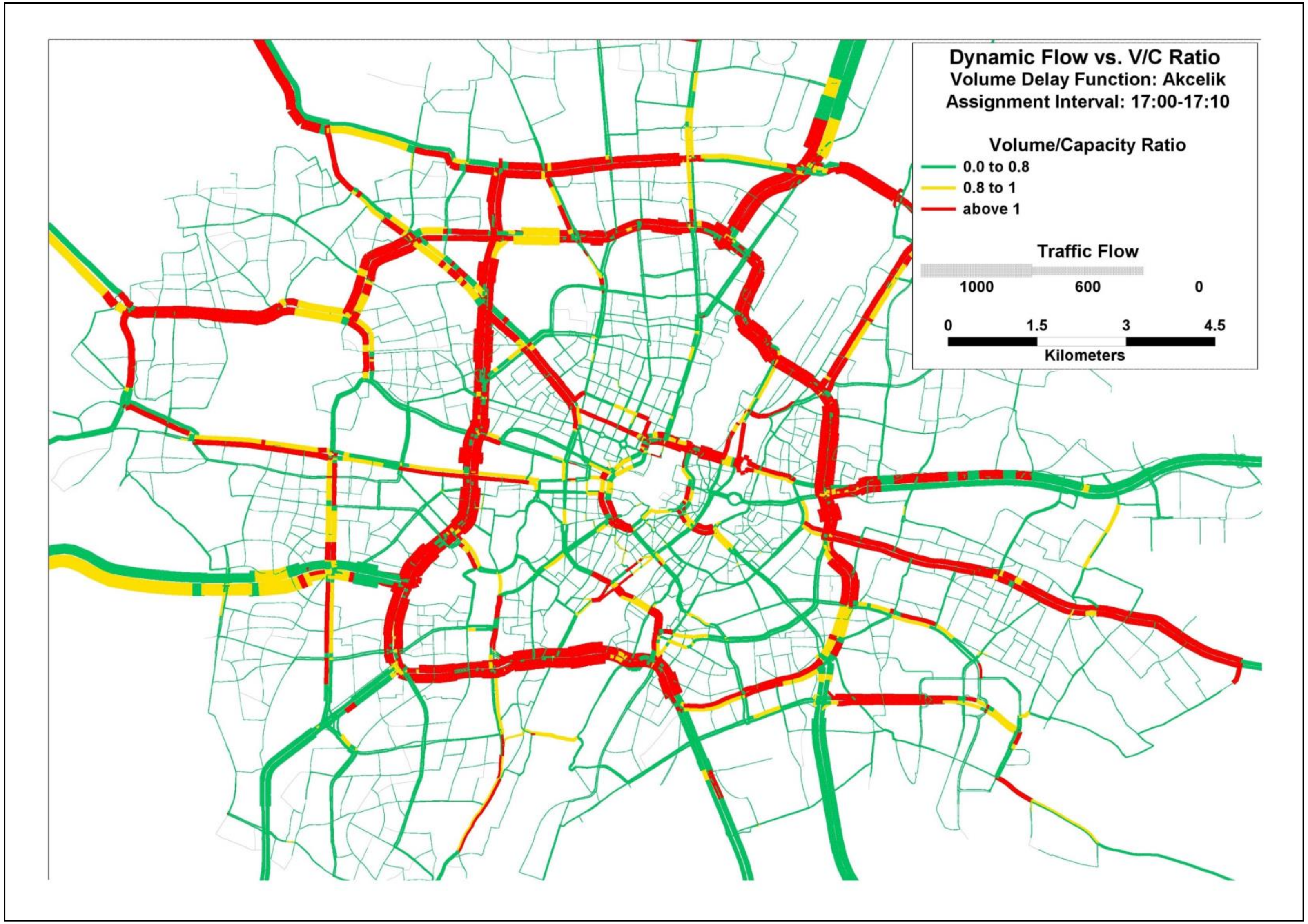
e.v) Assignment with **Akcelik** function for **morning** peak-hour **(7:40 – 7:50)**



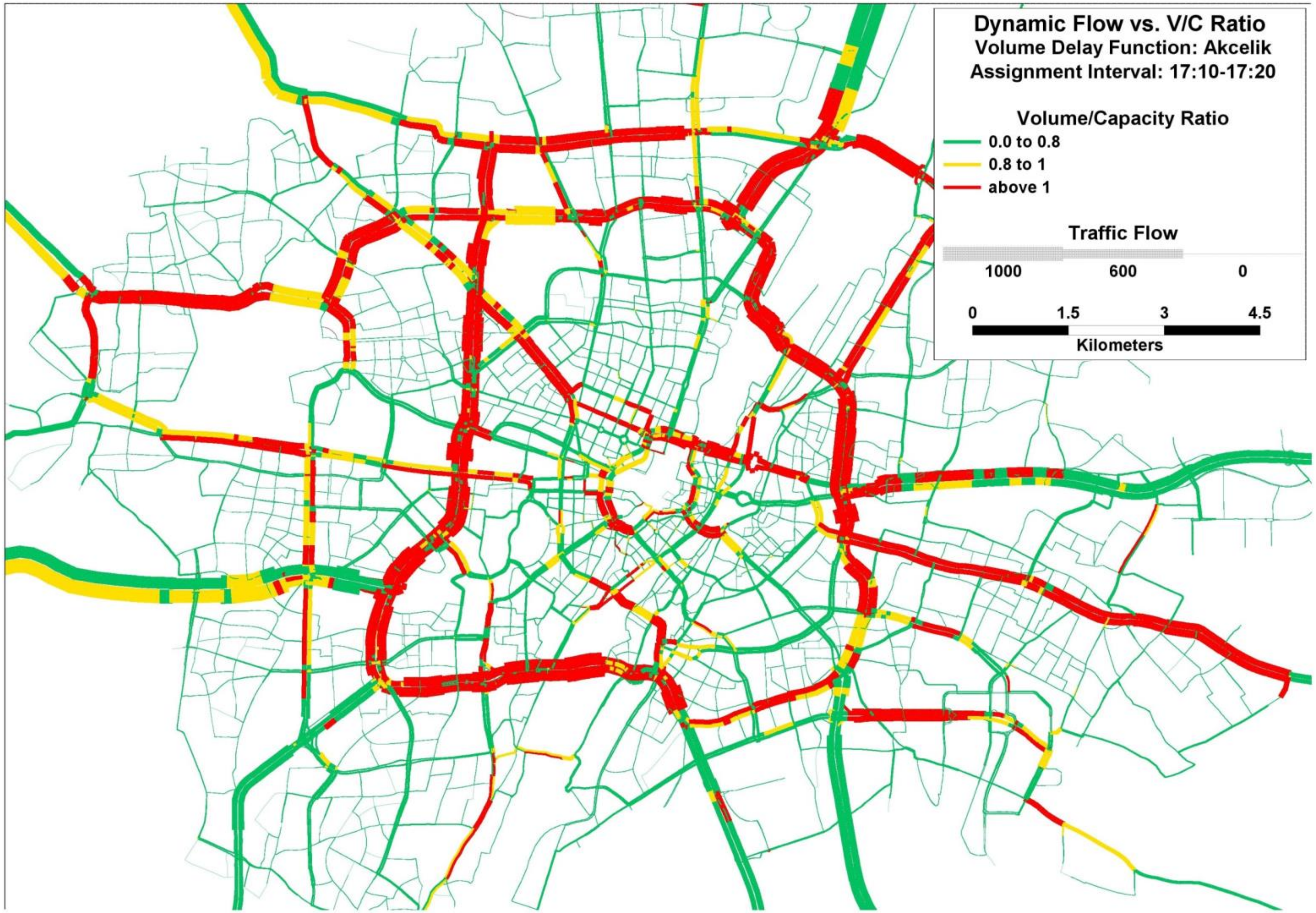
e.vi) Assignment with **Akcelik** function for **morning** peak-hour **(7:50 – 8:00)**



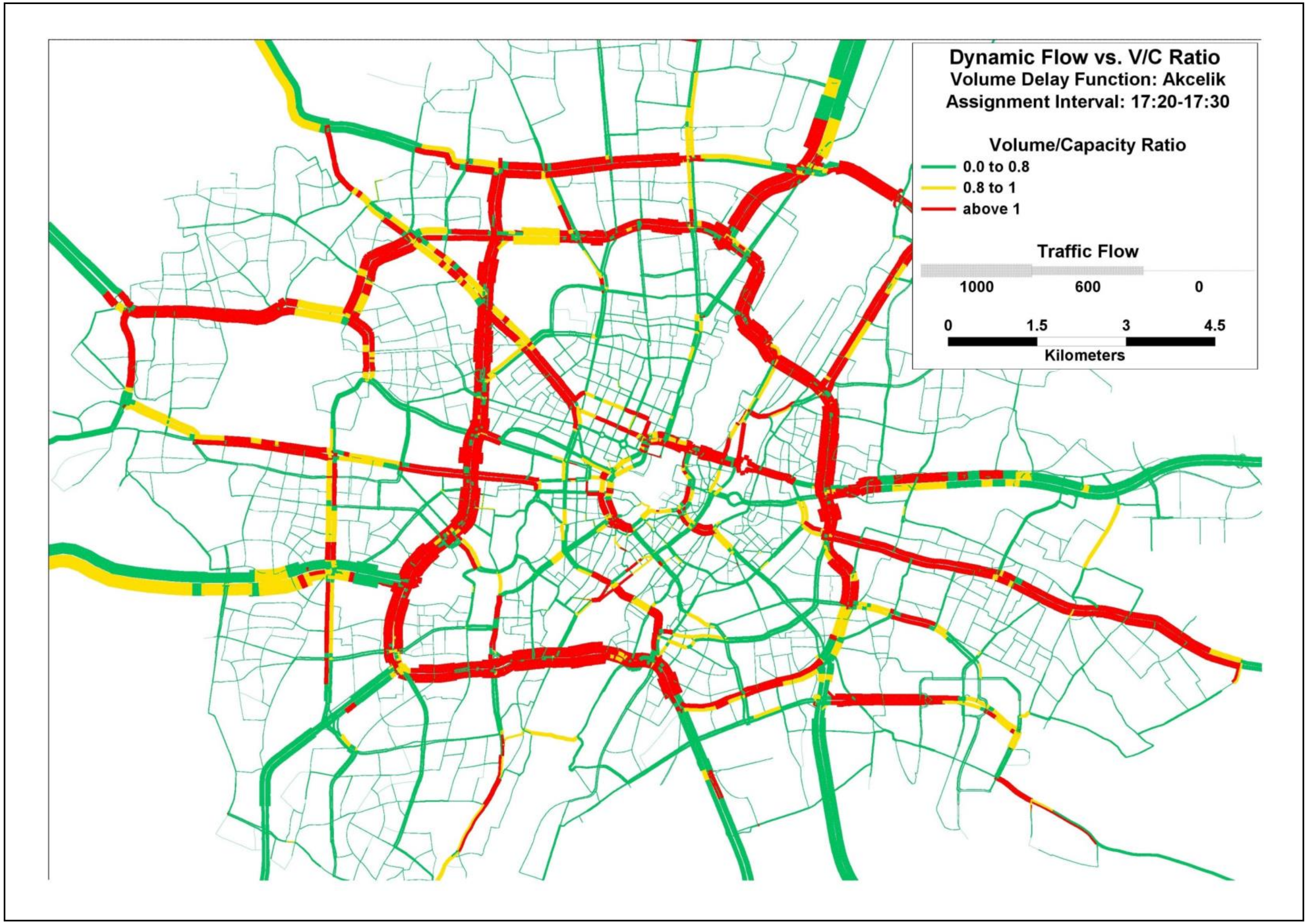
f.i) Assignment with **Akcelik** function for **afternoon** peak-hour **(17:00 – 17:10)**



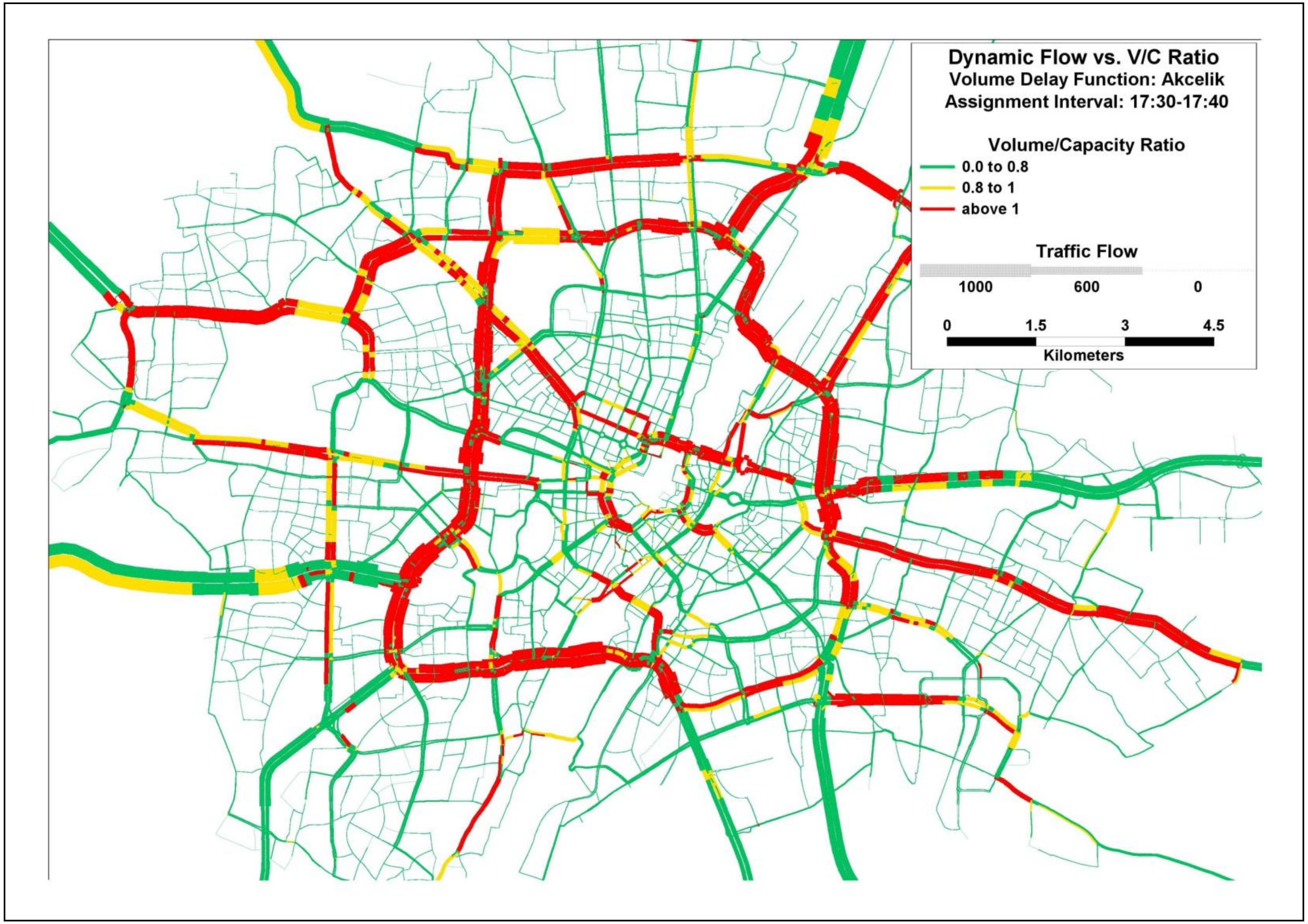
f.ii) Assignment with **Akcelik** function for **afternoon** peak-hour **(17:10 – 17:20)**



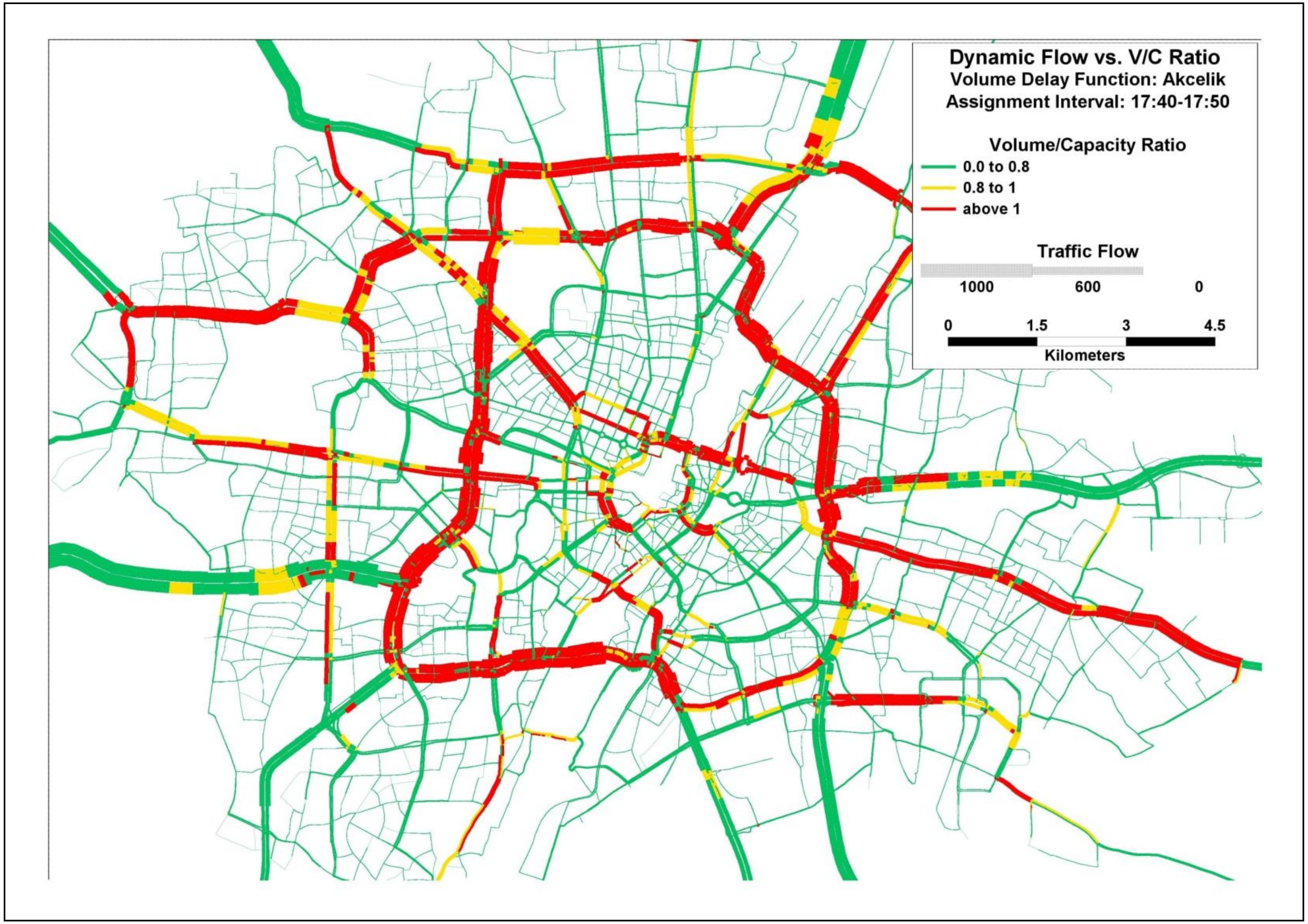
f.iii) Assignment with **Akcelik** function for **afternoon** peak-hour **(17:20 – 17:30)**



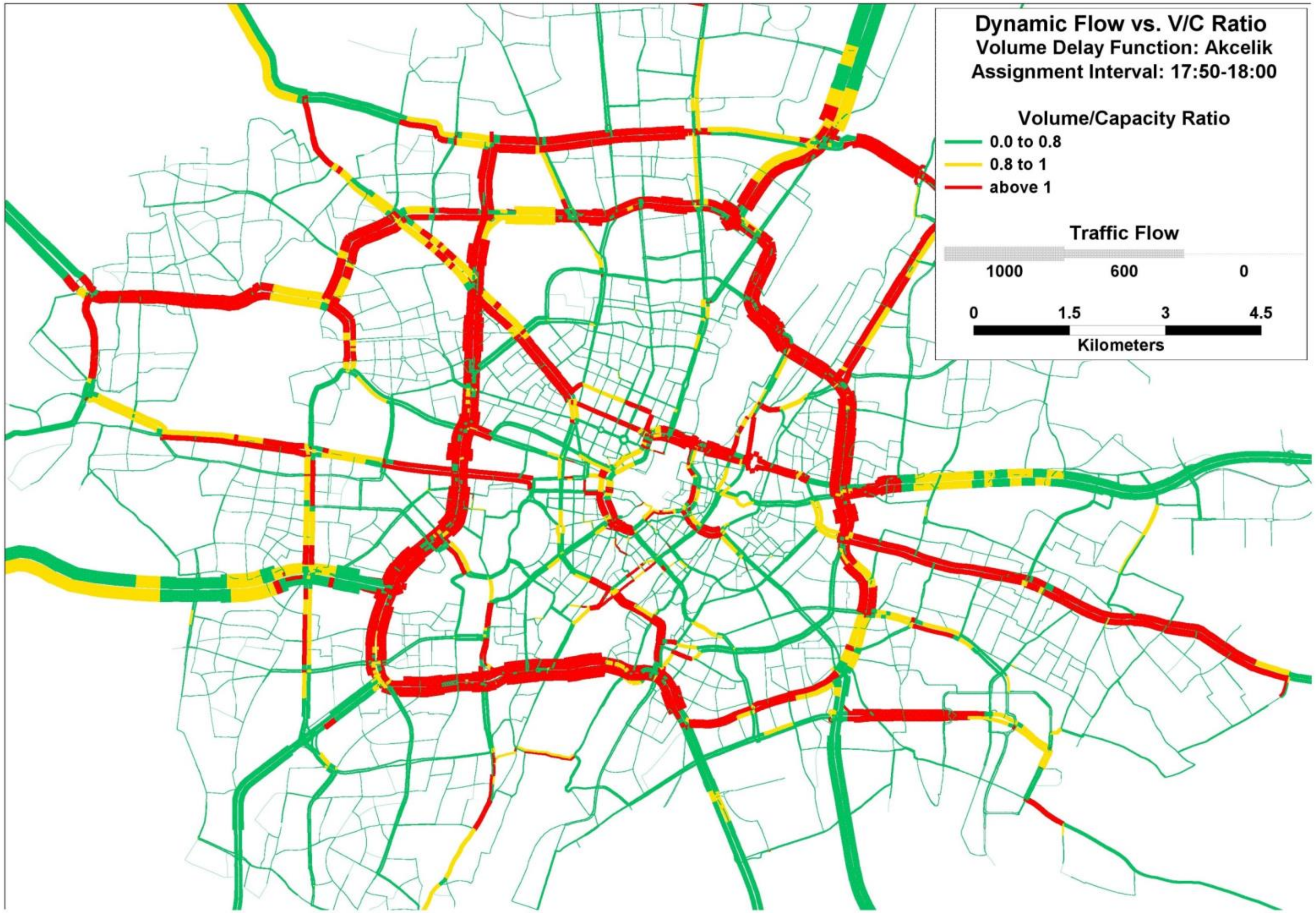
f.iv) Assignment with **Akcelik** function for **afternoon** peak-hour **(17:30 – 17:40)**



f.v) Assignment with **Akcelik** function for **afternoon** peak-hour **(17:40 – 17:50)**



f.vi) Assignment with **Akcelik** function for **afternoon** peak-hour **(17:50 – 18:00)**



B. Distribution of Average Assignment Speed

	BPR		Conical		Akcelik	
Free-flow Speed (km/h)	Average Assignment Speed (km/h)	Standard Deviation	Average Assignment Speed (km/h)	Standard Deviation	Average Assignment Speed (km/h)	Standard Deviation
10	9.07	1.00	8.13	1.68	7.07	3.01
25	23.76	1.66	22.73	2.50	19.79	5.09
40	34.98	5.10	33.10	6.11	24.42	10.76
50	39.38	8.84	38.51	10.21	30.23	14.93
60	41.78	12.26	40.02	13.30	36.54	19.80
70	43.00	17.02	40.83	17.15	35.70	21.74
80	74.02	13.78	70.35	11.90	76.72	8.66
100	97.89	12.95	91.26	14.74	97.59	9.06
110	109.98	0.12	103.06	5.61	105.79	3.59

C. Level of Congestion

V/C Ratio	Distance (km)			Percentage of Network (%)		
	BPR	Conical	Akcelik	BPR	Conical	Akcelik
0-0.25	8315.3	8450.7	9798.7	39	39	44
0.25-0.5	4653.1	4782.3	5776.9	22	22	26
0.5-0.75	3804.5	3892.1	3091.9	18	18	14
0.75-1	2841.0	3003.7	1665.4	13	14	7
1-1.25	1408.5	1455.7	1243.2	7	7	6
1.25-1.5	363.4	245.8	672.8	2	1	3
1.5-1.75	90.8	67.4	196.8	0	0	1
1.75-2	22.6	23.2	41.7	0	0	0
2-2.25	11.6	2.3	8.6	0	0	0
2.25-2.5	0.7	1.0	12.2	0	0	0
2.5-2.75	0.0	0.0	6.4	0	0	0
2.75-3	0.2	0.0	1.4	0	0	0

D. Assignment Volume vs. Traffic Count

Counting Location	Direction	Morning				Afternoon			
		Count	BPR	Conical	Akcelik	Count	BPR	Conical	Akcelik
1	AB	3433	2362	2352	2687	2290	2027	2003	2277
	BA	1477	1871	1842	2011	2569	1886	1843	2074
2	AB	3784	4380	4616	5229	3274	4428	4997	5671
	BA	2769	5038	4865	4984	4573	5414	5135	5373
3	AB	1438	2769	2701	2597	2246	2938	2950	2805
	BA	2458	3652	3623	3536	1977	2777	2783	2708
4	AB	1729	1987	1942	1911	2345	1933	1775	1772
	BA	2161	2298	2306	2413	1936	2035	2034	2107
5	AB	2673	3544	3509	3729	2858	3498	3487	3635
	BA	2931	2907	2837	3523	2815	2907	2879	3541
6	AB	3883	2911	2833	2670	2131	2182	2093	2030
	BA	1629	2562	2561	2755	3669	2252	2253	2347
7	AB	3586	3825	3761	3738	2437	2726	2620	2526
	BA	1786	2899	2902	3059	3282	2932	2906	3119
8	AB	5267	4714	4564	4496	3848	5225	5028	4790
	BA	3380	5902	5807	5966	4859	5212	5211	5517
9	AB	754	840	773	602	1711	1111	1163	1093
	BA	1930	1179	1220	1262	945	868	816	677
10	AB	1478	2877	2876	2876	1143	3312	3315	3314
	BA	1011	2454	3327	3215	1435	2966	2877	2719
11	AB	4997	4706	4719	4537	3205	5366	5392	5269
	BA	2498	5126	4996	4704	4791	5266	5053	4657
12	AB	3552	2429	2364	2252	1783	2155	2067	1978
	BA	1438	2718	2718	2718	3475	2121	2121	2121

E.Travel Time Analysis

Route Nr.	Origin Coordinates		Destination Coordinates		Morning					Afternoon				
					Google TT (min)		BPR TT (min)	Conical TT (min)	Akcelik TT (min)	Google TT (min)		BPR TT (min)	Conical TT (min)	Akcelik TT (min)
	X	Y	X	Y	Lowest	Highest				Lowest	Highest			
1	48.179110	11.515030	48.146190	11.615880	14	26	19.4	17.7	27.6	26	55	20.2	22.9	31.0
2	48.108670	11.520890	48.105880	11.620380	12	20	16.3	15.4	24.7	14	30	17.6	18.0	26.7
3	48.127980	11.622100	48.127700	11.502810	18	35	21.7	20.7	33.9	28	60	22.4	23.3	35.3
4	48.134090	11.583450	48.170840	11.481890	20	40	21.1	20.6	32.7	26	55	20.0	20.1	31.4
5	48.195260	11.572130	48.126030	11.536030	14	24	16.7	15.4	23.6	28	60	18.5	20.2	29.2
6	48.150900	11.557780	48.174420	11.635590	12	24	16.4	16.0	27.0	16	35	16.3	17.7	27.2
7	48.095130	11.523420	48.119560	11.600130	14	26	15.4	15.1	25.8	18	40	15.7	16.6	25.6
8	48.170230	11.573480	48.119320	11.548320	14	24	13.0	12.6	22.8	18	40	14.2	15.5	25.9
9	48.137640	11.672560	48.121570	11.564400	16	30	13.9	14.0	21.8	18	40	15.5	16.9	24.6
10	48.153990	11.537270	48.176660	11.589880	9	16	11.4	10.9	17.4	12	24	9.7	10.2	15.3
11	48.151780	11.612530	48.108130	11.612700	8	14	6.9	6.8	11.8	10	24	8.4	8.6	14.9
12	48.103370	11.605710	48.118070	11.515520	14	24	16.2	15.3	25.5	22	45	16.5	17.3	26.8
13	48.079914	11.488010	48.155030	11.505920	12	24	14.2	14.3	22.0	14	28	12.3	12.6	18.2
14	48.185900	11.622210	48.192416	11.509265	12	26	16.1	15.1	20.2	16	45	13.4	12.3	17.2
15	48.094140	11.653860	48.158660	11.642060	16	26	18.4	18.8	30.8	16	35	15.4	16.2	26.5