

# The meaning and limits of the fundamental diagram

**Dr. Ana Tsui Moreno Chou**

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## Learning outcomes

- a) Describe the relationships between the macroscopic variables speed, volume and density
- b) Define the types of traffic regimes according to the US Highway Capacity Manual
- c) List limitations of the fundamental diagram

## Recap

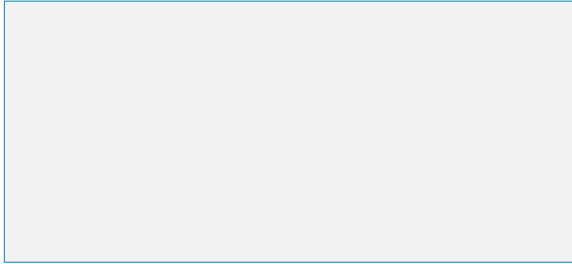
Basic flow parameters:

- **Volume  $q$** : the total number of vehicles passing over a given point or section of a lane or roadway during a given time interval
- **Density  $k$** : number of vehicles occupying a given length of a lane or roadway at a particular instant
- **Speed  $V$** : rate of motion expressed as distance per unit of time. ***Space mean speed*** is calculated as the average speed of vehicles traversing the segment at a particular instant

## Relationships

## Fundamental equation of traffic

The **fundamental equation of traffic** describes the general relationship between speed, volume and density as:

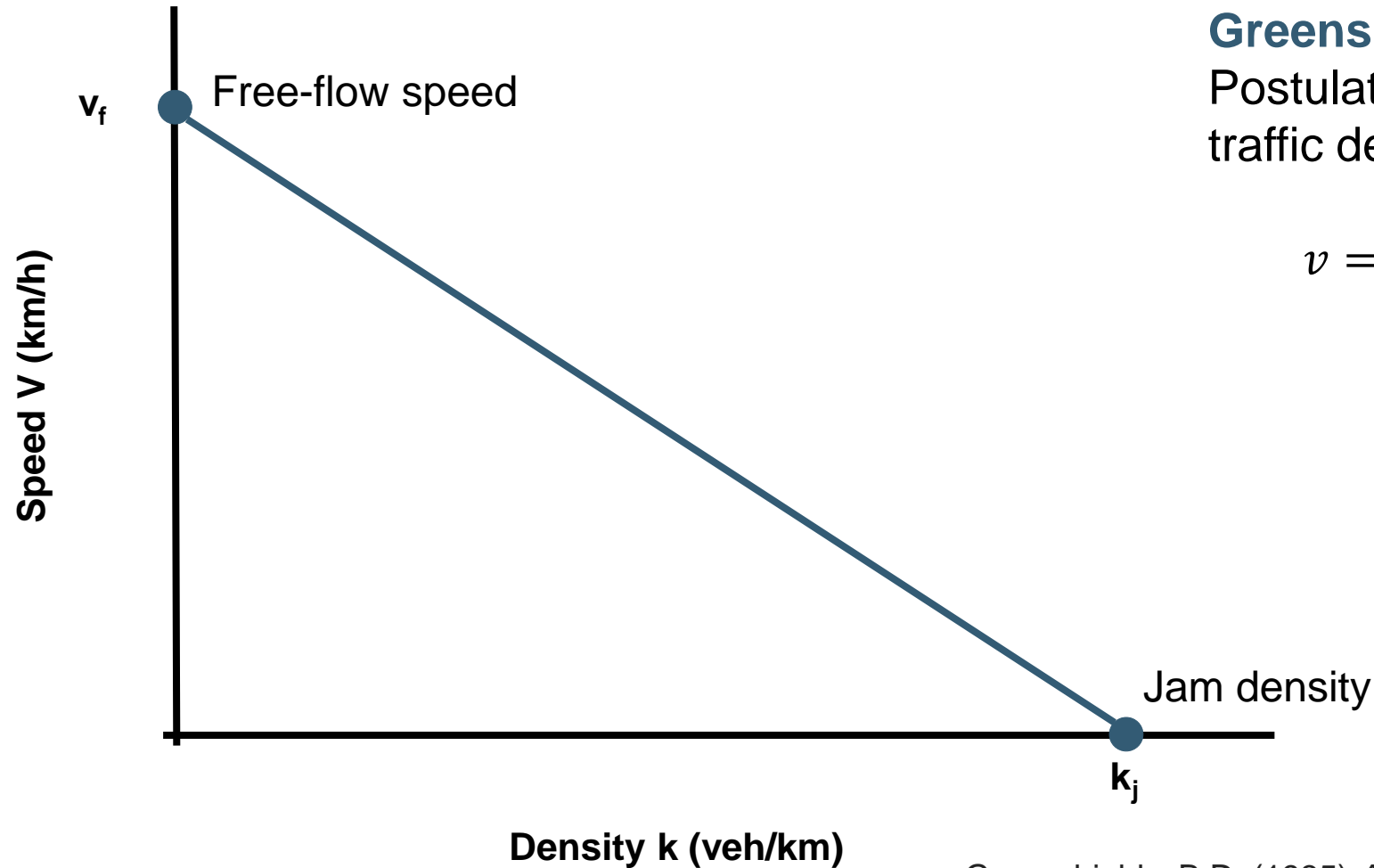


Where:  $q = \text{volume (veh/h)}$   
 $k = \text{density (veh/km)}$   
 $v = \text{speed (km/h)}$

This is valid for homogeneous, stationary traffic flow within each traffic regime

The  displays the fundamental equation of traffic

# Density and speed



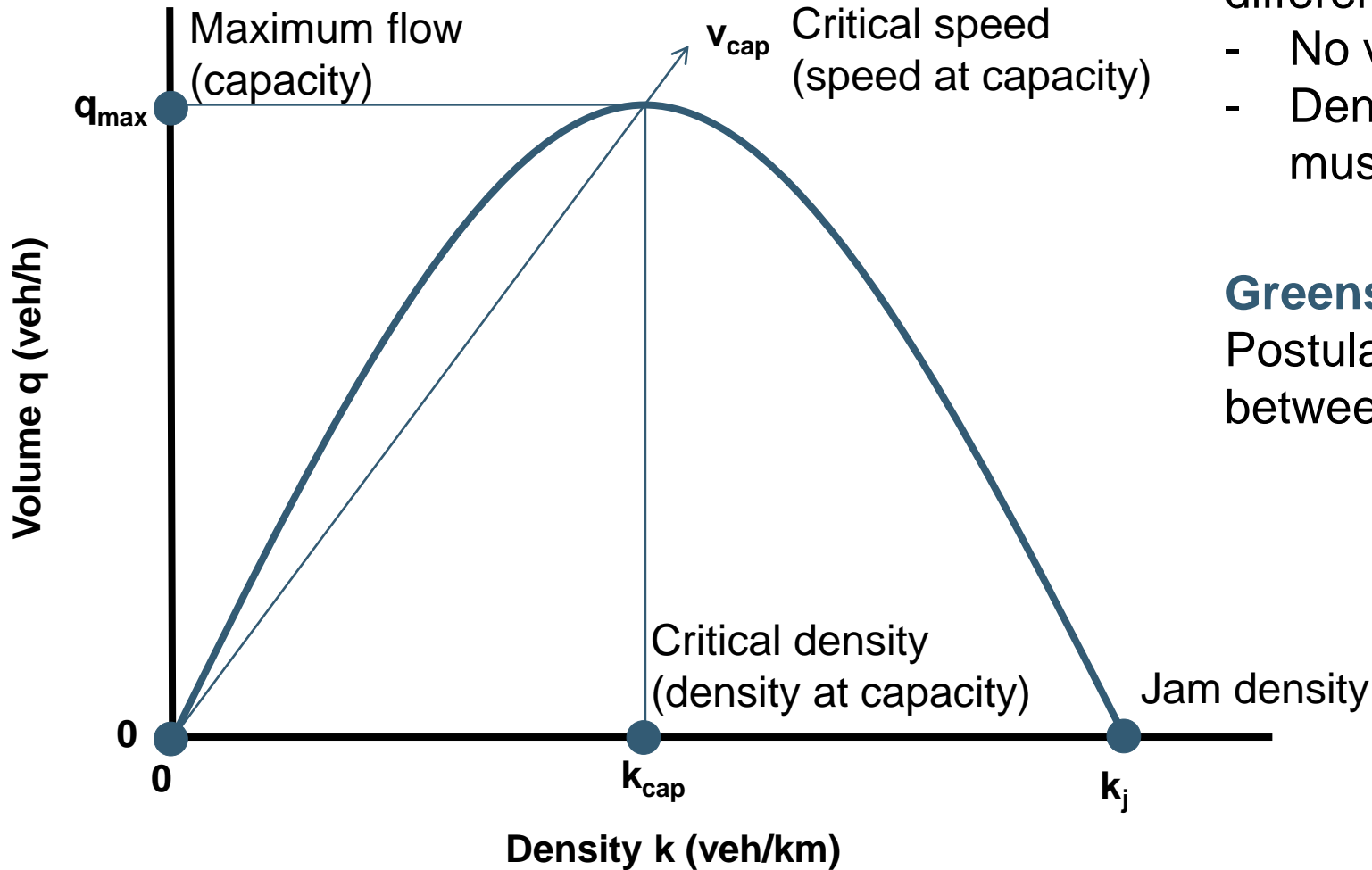
**Greenshields** (1935)

Postulated a **linear** relationship between traffic density and speed

$$v = v_f \left(1 - \frac{k}{k_j}\right)$$

Greenshields, B.D. (1935) *A Study of Traffic Capacity*. Highway Research Board, 14, 448-477.

# Density and volume



Zero volume occurs under two different conditions:

- No vehicles on the segment
- Density is so high that all vehicles must stop (jam density)

**Greenshields** (1935)

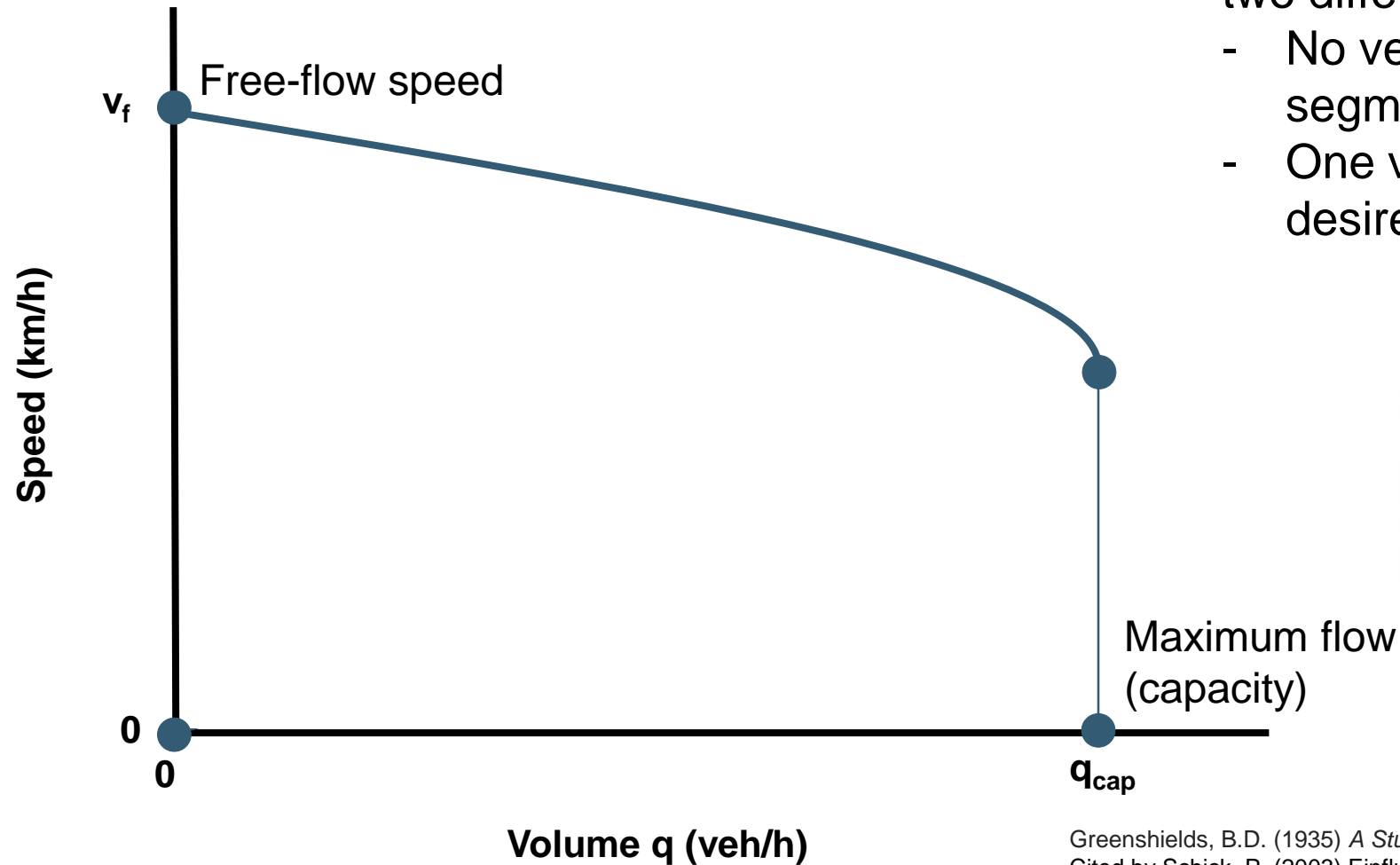
Postulated a **parabolic** relationship between density and volume

$$k_{\text{cap}} = \frac{k_j}{2}$$

$$q_{\text{cap}} = k_j \cdot v_{\text{cap}}$$

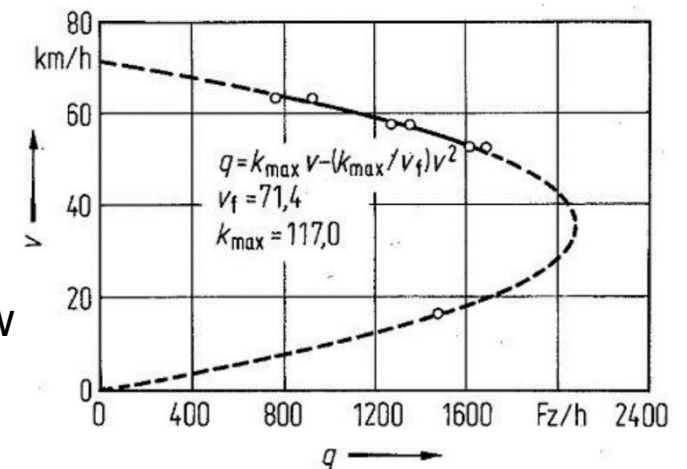
$$v_{\text{cap}} = \frac{v_f}{2}$$

# Volume and speed



Close to zero volume occurs under two different conditions:

- No vehicles moving on the segment
- One vehicle is moving at its desired speed (free-flow)



Greenshields, B.D. (1935) *A Study of Traffic Capacity*. Highway Research Board, 14, 448-477. Cited by Schick, P. (2003) Einfluss von Streckenbeeinflussungsanlagen auf die Kapazität von Autobahnabschnitten sowie die Stabilität des Verkehrsflusses. Dissertation, University of Stuttgart (Figure 3.3)



## Exercise

On a particular road, the Greenshields fundamental diagram holds with a free flow speed of 75 km/h and a jam density of 100 veh/km.

Does this determine the capacity with no further assumptions?

If so, calculate capacity.

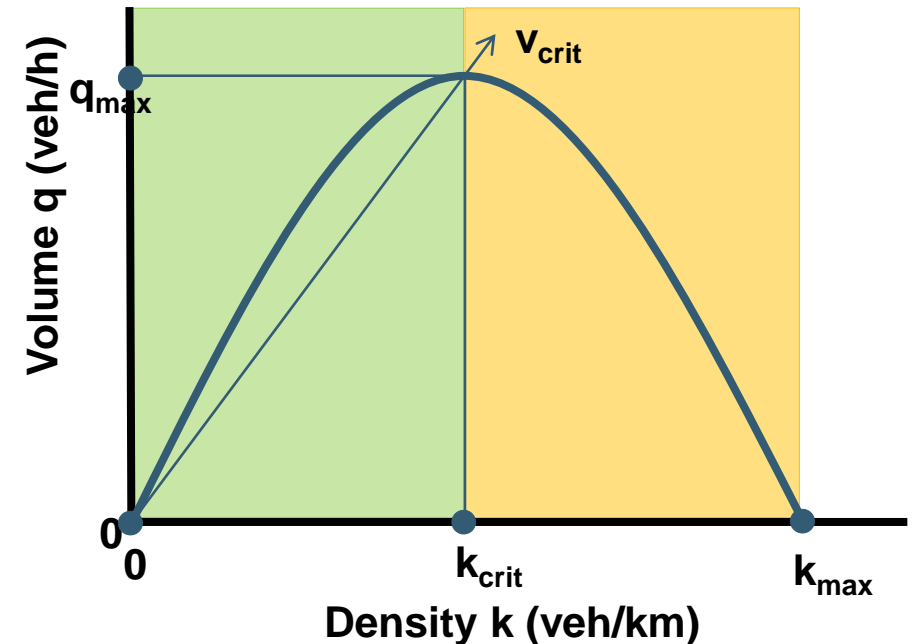
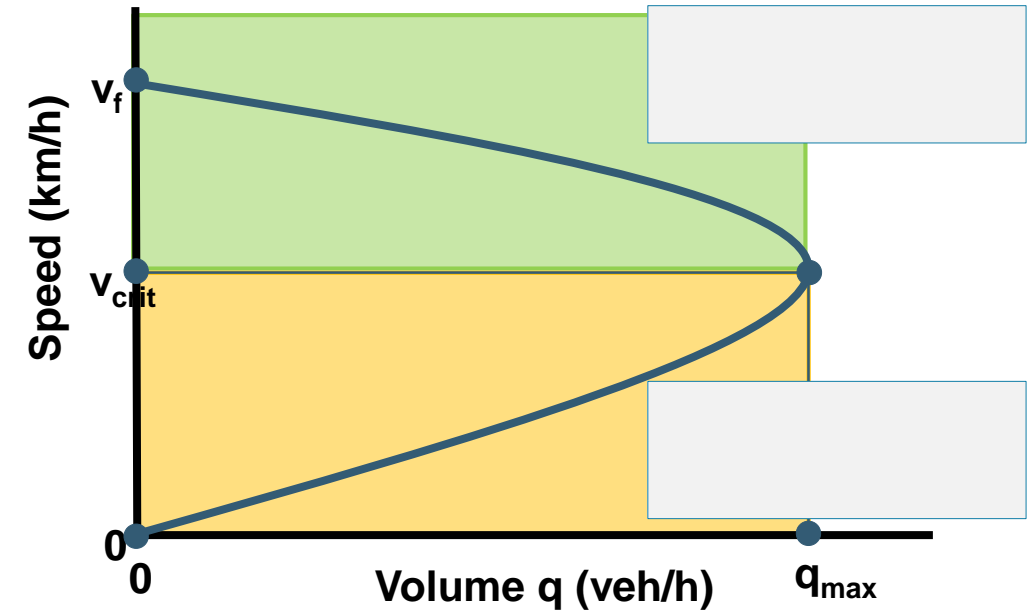
If not, explain why not.

## Traffic regimes

# Traffic regimes

There are two basic types of traffic regimes:

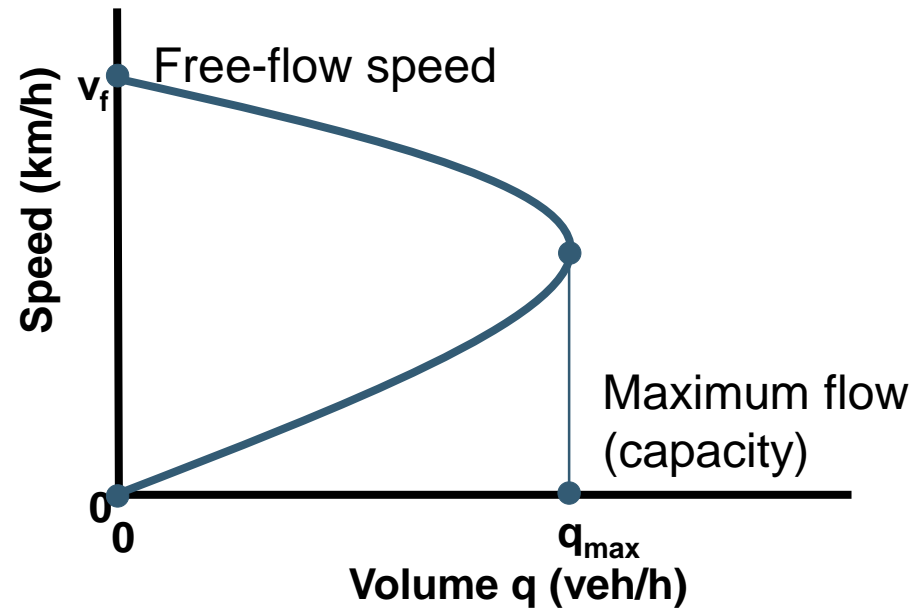
- 1) **Undersaturated flow:** Traffic flow where the volume is lower than the capacity
- 2) **Oversaturated flow:** Traffic flow where the volume exceeds the capacity



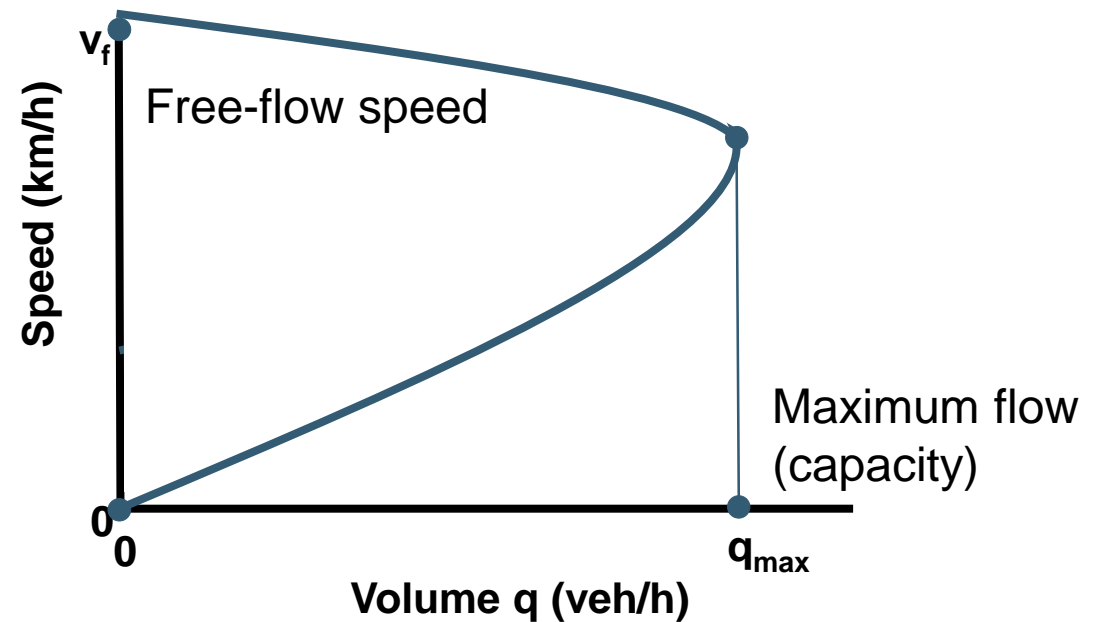
## Limitations

## Main limitations

- The form of the relationships depends on roadway conditions and on the segment length:
  - Different facilities will have different forms



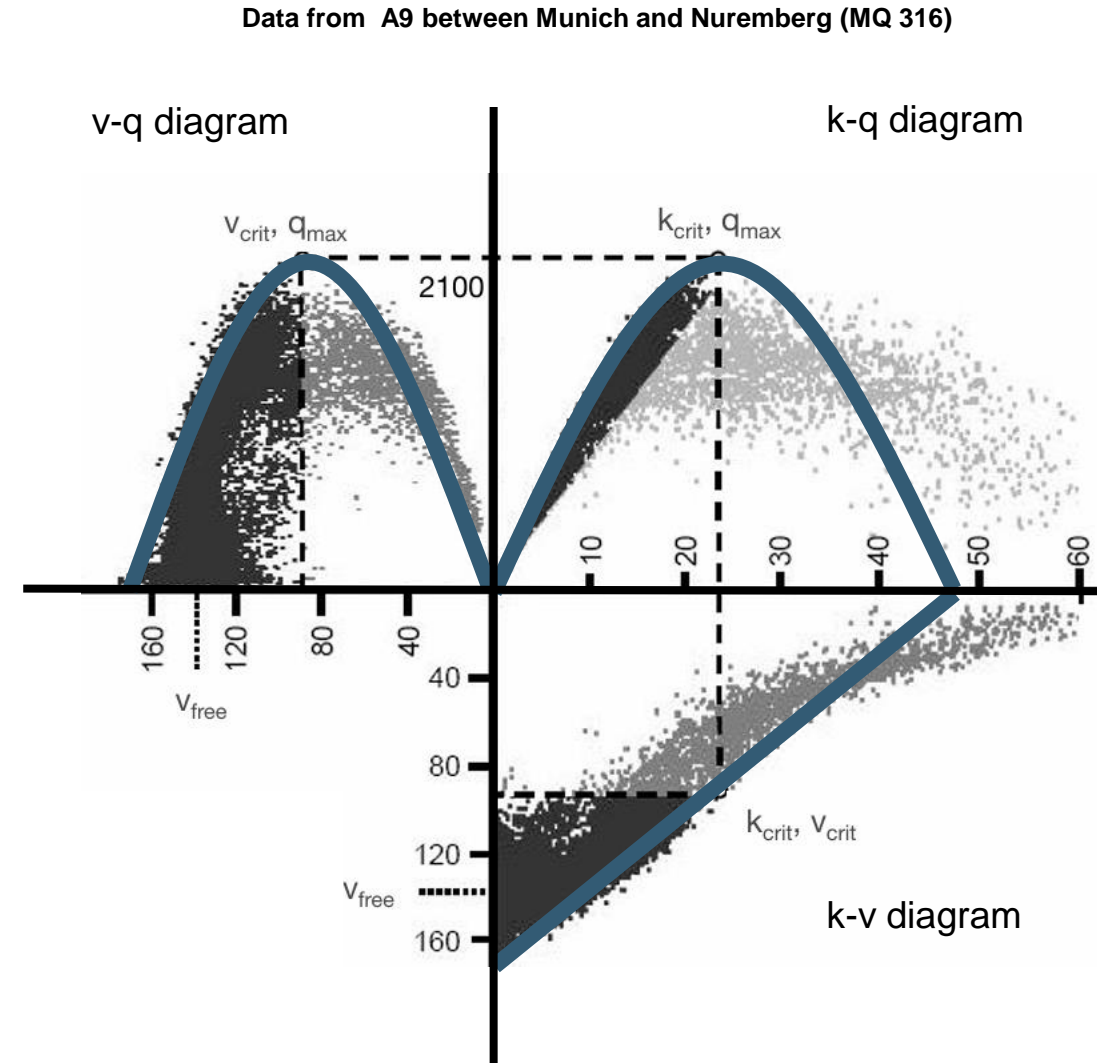
Facility: \_\_\_\_\_



Facility: \_\_\_\_\_

# Main limitations

- Traffic must be stationary and homogeneous:
  - Real-world data is affected by prevailing traffic or weather conditions
  - The idealized parabola may not be reached in freeway real-world data
  - Real-world data usually show discontinuities



Translated from Schick, P. (2003) Einfluss von Streckenbeeinflussungsanlagen auf die Kapazitaet von Autobahnabschnitten sowie die Stabilitaet des Verkehrsflusses. Dissertation, University of Stuttgart (Figure 3.1)

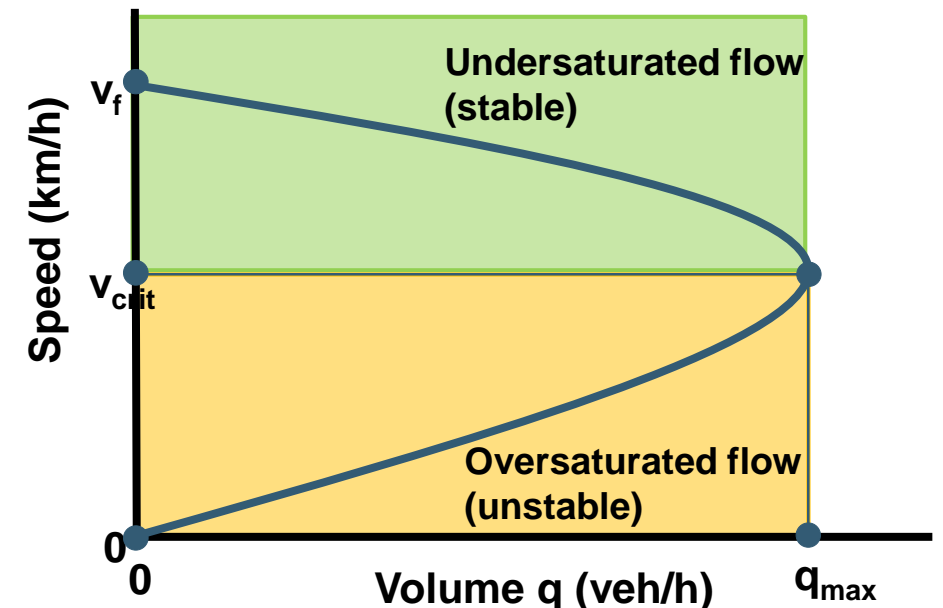
# Main limitations

- Traffic analysis is constrained to the analyzed period and facility:
  - A queue created from a prior breakdown of a facility may have not yet been dissipated
  - Traffic conditions may be affected by downstream conditions

The US Highway Capacity Manual (TRB, 2023) defines two types of traffic regimes:

- 1) **Undersaturated flow:** Traffic flow where (a) the arrival flow rate is lower than the capacity, (b) no residual queue remains from a prior breakdown of the facility, and (c) traffic flow is unaffected by downstream conditions
- 2) **Oversaturated flow:** Traffic flow where (a) the arrival flow rate exceeds the capacity, (b) a queue created from a prior breakdown of a facility has not yet dissipated, or (c) traffic flow is affected by downstream conditions

Source: Transportation Research Board (2023). Highway Capacity Manual 7.0



## Recap test

- ❑ Greenshields (1935) found that the relationship between traffic density and speed is:
  - a) Parabolic
  - b) Linear
  - c) Log-linear
  
- ❑ According to the basic equation of traffic states, volume ( $q$ ) is equal to:
  - a) Speed divided by density
  - b) Density divided by speed
  - c) Speed times density
  
- ❑ The US Highway Capacity Manual defines oversaturated flow conditions when:
  - a) Traffic flow is affected by downstream conditions
  - b) The arrival flow rate is lower than the capacity
  - c) No residual queue remains from a prior breakdown



# The meaning and limits of the fundamental diagram

**Dr. Ana Tsui Moreno Chou**

**[ana.moreno@tum.de](mailto:ana.moreno@tum.de)**

**Further materials**

## Take-home exercise

Describe qualitatively and quantitatively the macroscopic parameters:

- Traffic density  $k$ ,
- Traffic volume  $q$  and
- Average velocity  $v_m$

On a two-lane rural highway with good weather conditions and adverse weather conditions for the traffic states

- **Undersaturated and**
- **Oversaturated.**

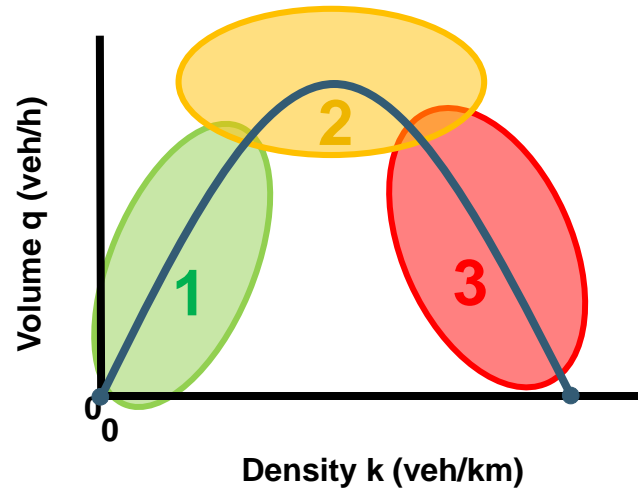
Plot your results in the fundamental diagrams and deduce relationships between these parameters on the two weather conditions.

# Traffic regimes

## Other examples

May (1990) defines three types of traffic regimes:

- 1) **Free flow**: high speed with low traffic flow and density
- 2) **Impeded free flow**: maximum traffic flow at optimum (critical) density and speed
- 3) **Impeded flow**: high traffic density with low traffic flow and speed

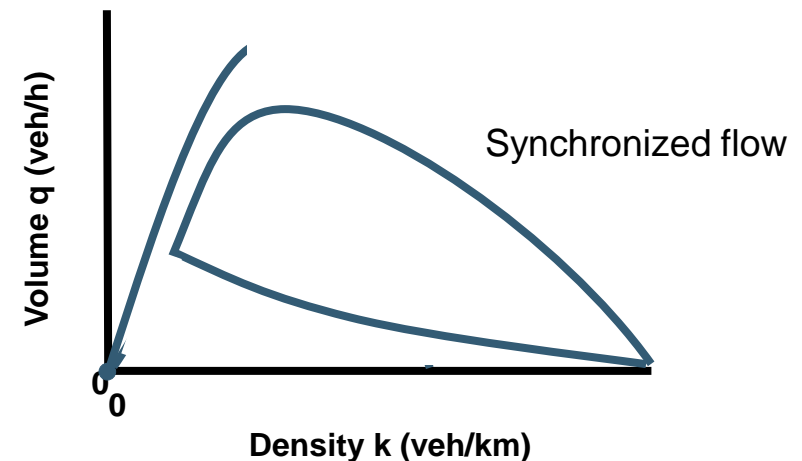


May, A. D., (1990). *Traffic Flow Fundamentals*. Prentice Hall, Englewood Cliffs, N.J.

Fundamental diagram

Kerner (2004) defines two areas and three phases:

- 1) **Free flow**: no interrelation between vehicles, overtaking possible at any time
- 2) **Synchronized flow**: occurs behind a bottleneck. Few stops, comparable speed on lanes
- 3) **Wide moving jam**: front-end and rear-end of jam are described by shock waves against the traffic direction



Kerner, B. Kerner, (2004) Three-phase traffic theory and highway capacity, *Physica A: Statistical Mechanics and its Applications* 333, 379-440.

## Applications of the fundamental diagram theory

- Design of road sections (e.g. determination of capacity)
- Traffic control (e.g. optimizing stable flow conditions)
- Traffic flow simulation (e.g. macroscopic models)
- Traffic state estimation (e.g. congestion propagation)