2nd German Robotics Conference
March 11-13, 2026. Cologne, Germany

FBGA: a Forward-Backward Method for Online Time-Optimal Velocity Planning with Generic Acceleration Constraints

Mattia Piazza[1], Mattia Piccinini[2], Sebastiano Taddei[1,3], Francesco Biral[1], and Enrico Bertolazzi[1]

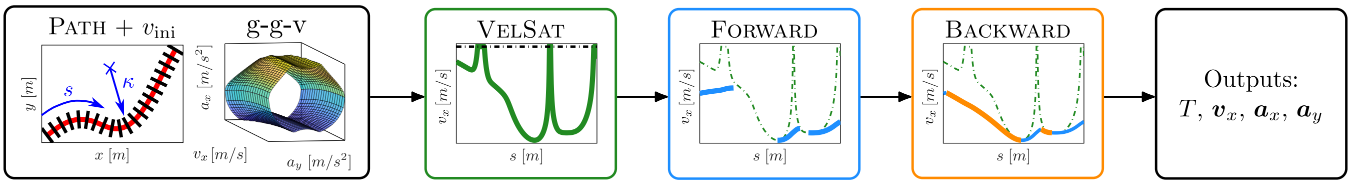


Fig. 1. Main phases of our FBGA method. Inputs are: vectors of curvilinear abscissas $\boldsymbol{s}$ and curvatures $\boldsymbol{\kappa}$ defining the path, initial speed v_{ini}, and the g-g-v acceleration constraints. Outputs are: speed and acceleration profiles $\{\boldsymbol{v}_x, \boldsymbol{a}_x, \boldsymbol{a}_y\}$, and maneuver time T. Taken from [1] ©2025 IEEE.

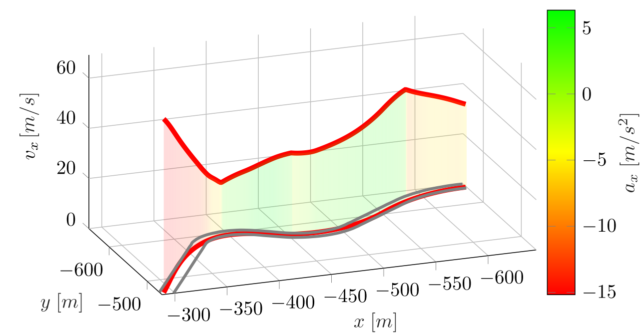
***Abstract*— We present FBGA, a new algorithm for time-optimal velocity planning under generic acceleration constraints. By extending previous forward-backward approaches to handle custom acceleration constraints, our FBGA matches the accuracy of optimal control baselines while being up to three orders of magnitude faster. Our open-source C++ implementation is available at: `https://github.com/DRIVEWISE/FBGA`.**

I. Introduction

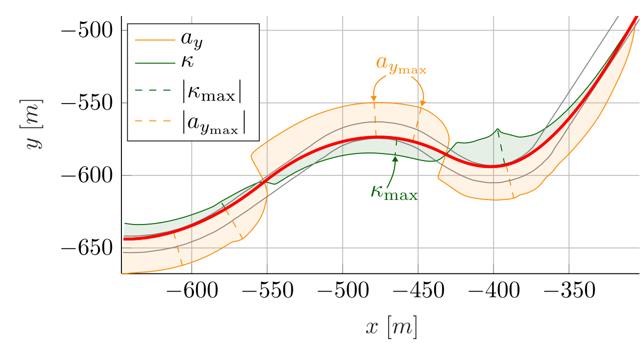
Time-optimal velocity planning along given paths is fundamental for autonomous racing [2], [3], drone flight [4], and robot navigation [5]. In dynamic environments, online velocity optimization is critical, as many candidate maneuvers must be evaluated efficiently during planning [2], [6]. Existing minimum-time velocity planners face a trade-off between accuracy and computational efficiency. Optimal Control Problems (OCPs) [7] and Quasi-Steady-State (QSS) approaches [8], [9] achieve high accuracy and can handle complex, speed-dependent acceleration limits (g-g-v diagrams). However, their high computational cost prevents online multi-query planning or requires heuristic apex selection (Fig. 2). Conversely, semi-analytical Forward-Backward (FB) and sequential methods [10], [11] are computationally efficient but restricted to conservative box-shaped acceleration bounds. Accurate performance modeling instead requires complex g-g-v diagrams (Fig. 1) that capture the speed-dependent coupling between the longitudinal and lateral accelerations [8], [12]–[14].

Our approach achieves the accuracy of optimal control methods while being up to three orders of magnitude faster, enabling online use within sampling-based trajectory planners. Our work, published in [1], proposes the following:

1) FBGA: a new method for time-optimal speed planning on a given path with custom acceleration constraints.



(a) Path, velocity (v_x) and longitudinal acceleration (a_x).



(b) Lateral acceleration (a_y) and curvature (κ), with their local maxima.

Fig. 2. (a) Time-optimal velocity and longitudinal acceleration profiles on the corners 1 and 2 of the Catalunya circuit. (b) Lateral acceleration and curvature profiles without apex computation. Taken from [1] ©2025 IEEE.

2) Validation on different racetracks and vehicle types.
3) Comparison against OCPs, showing the same level of accuracy with 1000x computational speed-up.

II. Methodology

We introduce a new FB method capable of handling generic acceleration constraints, represented by complex g-g-v diagrams (Fig. 1). In our algorithm, reported in [1], we exploit custom non-derivative root finding methods to efficiently compute the maximum feasible longitudinal acceleration at each point of the path, given the corresponding lateral acceleration and the g-g-v constraints (Fig. 1). Further details are provided in [1].

[1] Department of Industrial Engineering, University of Trento, 38123 Trento, Italy, `name.surname@unitn.it`.
[2] Professorship of Autonomous Vehicle Systems, Technical University of Munich, 85748 Garching, Germany; Munich Institute of Robotics and Machine Intelligence (MIRMI), `mattia.piccinini@tum.de`.
[3] Department of Electrical and Information Engineering, Politecnico di Bari, 70125 Bari, Italy, `s.taddei@phd.poliba.it`.

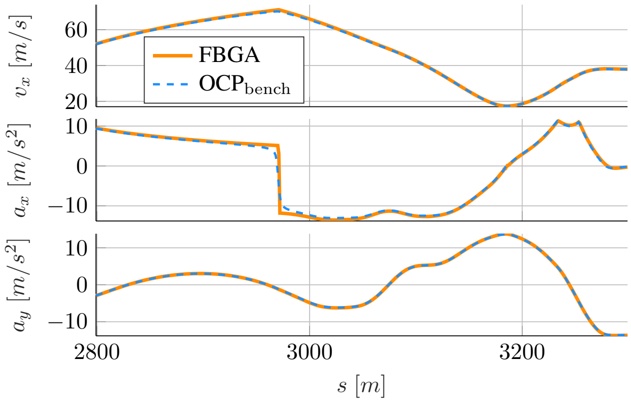
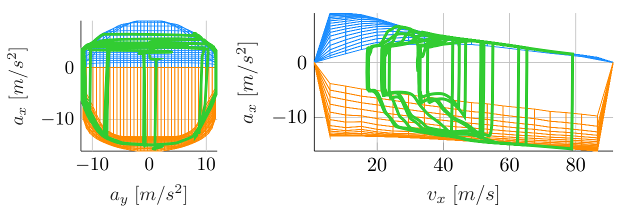


Fig. 3. Velocity, longitudinal and lateral acceleration profiles of our FBGA and the benchmark OCP$_{\text{bench}}$, for a racing car (a) and motorcycle (b) at turn 9 of the Sepang circuit. Taken from [1] ©2025 IEEE.



(a) Top view

(b) Lateral view

Fig. 4. g-g-v envelope (blue-orange lines) for the race car model, and solution computed by our FBGA (green line) on the Catalunya circuit.

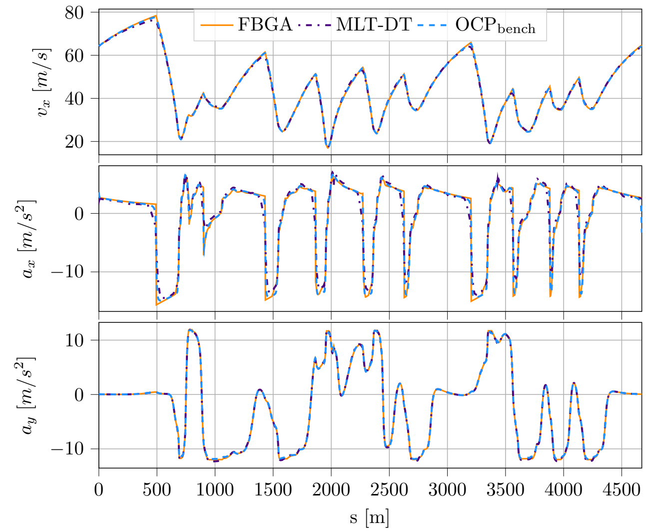
III. Results

We validate FBGA on five racetracks (Table I) and with two vehicle types: a racing car (g-g-v taken from [7]) and a racing motorcycle (g-g-v derived from [15] and notably non-convex). Benchmarking is performed against the OCP in [13], named OCP$_{\text{bench}}$, solved with the PINS solver [16].

A. Maneuver Analysis, Lap and CPU Times

Fig. 3 shows the resulting speed and acceleration profiles at the turn 9 of the Sepang circuit, with the motorcycle model. FBGA yields results very close to the benchmark OCP$_{\text{bench}}$, with minor deviations at the acceleration-braking transitions. Fig. 4 depicts the g-g-v envelope along with the FBGA solution (green line) on the Catalunya circuit, with the race car model. The solution adheres closely to the g-g-v envelope, satisfying the constraints within the solver's precision. Furthermore, we compare FBGA against a minimum-lap-time OCP solved with a high-fidelity double-track vehicle model (MLT-DT), taken from [17] and including an MF5.2 tire model, real engine torque curves, and a limited-slip differential. As shown in Fig. 5, FBGA matches MLT-DT and OCP$_{\text{bench}}$, with maximum lap time differences of 0.2 s.

Table I shows that FBGA differs from OCP$_{\text{bench}}$ by only 0.11%–0.36% in lap time, which is comparable to variations among professional race drivers [18]. FBGA slightly underestimates the lap times due to its assumption of unbounded longitudinal jerk. Nonetheless, this assumption enables fast computations using our FB scheme, making it suitable for real-time applications.



FBGA	MLT-DT	OCP$_{\text{bench}}$
113.96 s	114.05 s	114.16 s

Fig. 5. Comparing our FBGA against the OCP$_{\text{bench}}$ and MLT-DT benchmarks, on the Catalunya circuit.

Circuit	Vehicle	Lap time [s]		CPU time [ms]	
		OCP$_{\text{bench}}$	**FBGA**	OCP$_{\text{bench}}$	**FBGA**
Catalunya	Car	112.461	112.204	8017.15	9.86
	Moto	105.381	104.999	693.91	3.31
Valencia	Car	104.742	104.520	7411.99	8.85
	Moto	96.752	96.434	830.82	3.40
Misano	Car	107.740	107.451	6805.87	9.66
	Moto	98.814	98.497	602.19	3.37
Sepang	Car	135.480	135.086	8239.47	11.44
	Moto	125.483	125.034	1282.58	4.30
Palm Beach	Car	79.963	79.869	4819.43	6.38
	Moto	74.790	74.537	706.43	2.39

TABLE I

Comparison of lap times and CPU times between our FBGA and the benchmark OCP$_{\text{bench}}$, both solved with the same mesh, on an M2 Max Apple Silicon chip.

The CPU times in Table I highlight the efficiency of FBGA compared to OCP$_{\text{bench}}$. FBGA computes full-lap speed profiles in $2.39 - 11.44$ ms, which is $2 - 3$ orders of magnitude faster than OCP$_{\text{bench}}$. The CPU times are heavily influenced by the path length and constraint complexity, as we show in [1]. These results demonstrate FBGA's suitability for real-time trajectory planning in dynamic environments.

IV. Conclusions

We introduced FBGA, a novel method for time-optimal velocity planning that supports generic acceleration constraints. Validated on car and motorcycle models, FBGA matches the lap times of optimal control baselines within 0.36%, while being up to three orders of magnitude faster. This efficiency makes FBGA ideal for real-time applications, such as warm-starting complex OCPs or serving as a high-fidelity building block for online sampling-based planners in autonomous racing.

REFERENCES

[1] M. Piazza, M. Piccinini, S. Taddei, F. Biral, and E. Berolazzi, "Real-time velocity profile optimization for time-optimal maneuvering with generic acceleration constraints," *IEEE Robotics and Automation Letters*, vol. 11, no. 2, pp. 1674–1681, 2026.

[2] L. Ögretmen, M. Rowold, A. Langmann, and B. Lohmann, "Sampling-based motion planning with online racing line generation for autonomous driving on three-dimensional race tracks," in *2024 IEEE Intelligent Vehicles Symposium (IV)*, 2024, pp. 811–818.

[3] M. Piccinini, S. Gottschalk, M. Gerdts, and F. Biral, "Computationally efficient minimum-time motion primitives for vehicle trajectory planning," *IEEE Open Journal of Intelligent Transportation Systems*, vol. 5, pp. 642–655, 2024.

[4] A. Romero, R. Penicka, and D. Scaramuzza, "Time-optimal online replanning for agile quadrotor flight," *IEEE Robotics and Automation Letters*, vol. 7, no. 3, pp. 7730–7737, 2022.

[5] M. Benko Loknar, G. Klančar, and S. Blažič, "Minimum-time trajectory generation for wheeled mobile systems using bézier curves with constraints on velocity, acceleration and jerk," *Sensors*, vol. 23, no. 4, 2023.

[6] M. Piazza, M. Piccinini, S. Taddei, and F. Biral, "MPTree: A sampling-based vehicle motion planner for real-time obstacle avoidance," *IFAC-PapersOnLine*, vol. 58, no. 10, pp. 146–153, 2024.

[7] M. Piccinini, S. Taddei, E. Pagot, E. Bertolazzi, and F. Biral, "How optimal is the minimum-time manoeuvre of an artificial race driver?" *Vehicle System Dynamics*, vol. 63, no. 12, pp. 2213–2240, 2025.

[8] M. Veneri and M. Massaro, "A free-trajectory quasi-steady-state optimal-control method for minimum lap-time of race vehicles," *Vehicle System Dynamics*, vol. 58, no. 6, pp. 933–954, 2020.

[9] F. Rodegher, S. Lovato, and M. Massaro, "The effect of load-dependent tyre friction and aerodynamic cross-flow forces on the gg maps and minimum-lap-time of motorcycles," *Vehicle System Dynamics*, pp. 1–19, 2024.

[10] M. Frego, E. Bertolazzi, F. Biral, D. Fontanelli, and L. Palopoli, "Semi-analytical minimum time solutions with velocity constraints for trajectory following of vehicles," *Automatica*, vol. 86, pp. 18–28, 2017.

[11] L. Consolini, M. Locatelli, and A. Minari, "A sequential algorithm for jerk limited speed planning," *IEEE Transactions on Automation Science and Engineering*, vol. 19, no. 4, pp. 3192–3209, 2022.

[12] D. Limebeer, M. Massaro, and A. Dollar, "Asymmetric vehicle performance assessment using gg diagrams," *Vehicle System Dynamics*, pp. 1–27, 2024.

[13] M. Piccinini, S. Taddei, M. Piazza, and F. Biral, "Impacts of ggv constraints formulations on online minimum-time vehicle trajectory planning," *IFAC-PapersOnLine*, vol. 58, no. 10, pp. 87–93, 2024.

[14] F. Werner, S. Sagmeister, M. Piccinini, and J. Betz, "A quasi-steady-state black box simulation approach for the generation of g-g-g-v diagrams," in *2025 IEEE Intelligent Vehicles Symposium (IV)*, 2025, pp. 2503–2509.

[15] F. Biral and R. Lot, "An interpretative model of g-g diagrams of racing motorcycle," in *Proceedings of the 3rd ICMEM International Conference on Mechanical Engineering and Mechanics*, Beijing, China, 2009.

[16] F. Biral, E. Bertolazzi, and P. Bosetti, "Notes on numerical methods for solving optimal control problems," *IEEJ Journal of Industry Applications*, vol. 5, no. 2, pp. 154–166, 2016.

[17] E. Pagot, "A study of a robust and accurate framework for minimum-time optimal control of high-performance cars: from coaching professional drivers to autonomous racing," Ph.D. dissertation, University of Trento, Italy, 2023.

[18] J. C. Kegelman, L. K. Harbott, and J. C. Gerdes, "Insights into vehicle trajectories at the handling limits: analysing open data from race car drivers," *Vehicle System Dynamics*, vol. 55, no. 2, pp. 191–207, 2017.